

# Meson mixing and lifetimes



Matthew Kirk



LHCb-UK 2018

based on **1711.02100** (MK, Rauh, Lenz)

**1712.06572** (Di Luzio, MK, Lenz)

# Status before 2017 – experiment

- $B_s$  Mixing
  - $\Delta M_s$  is extremely well measured (0.1% uncertainty)
  - $\Delta \Gamma_s$  known with sub 10% uncertainty
- B lifetime ratios
  - $\tau(B_s)/\tau(B_d)$  known with < 0.25% uncertainty
- D Mixing
  - First > 5 sigma measurement from LHCb in 2012
  - $O(10\%)$  accuracy
- D lifetime ratios
  - $\tau(D^+)/\tau(D^0)$  known with <1% uncertainty

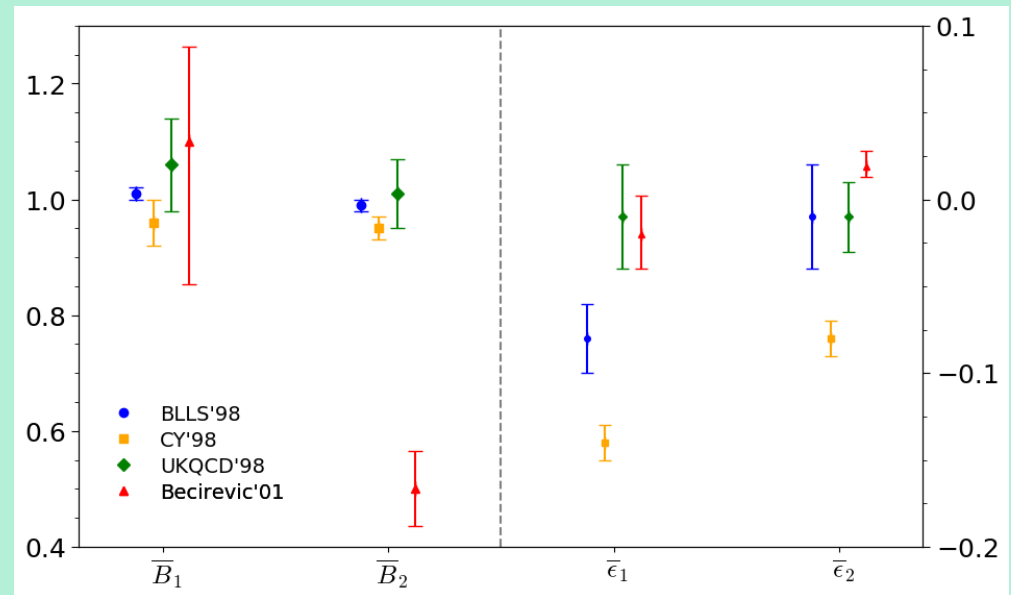
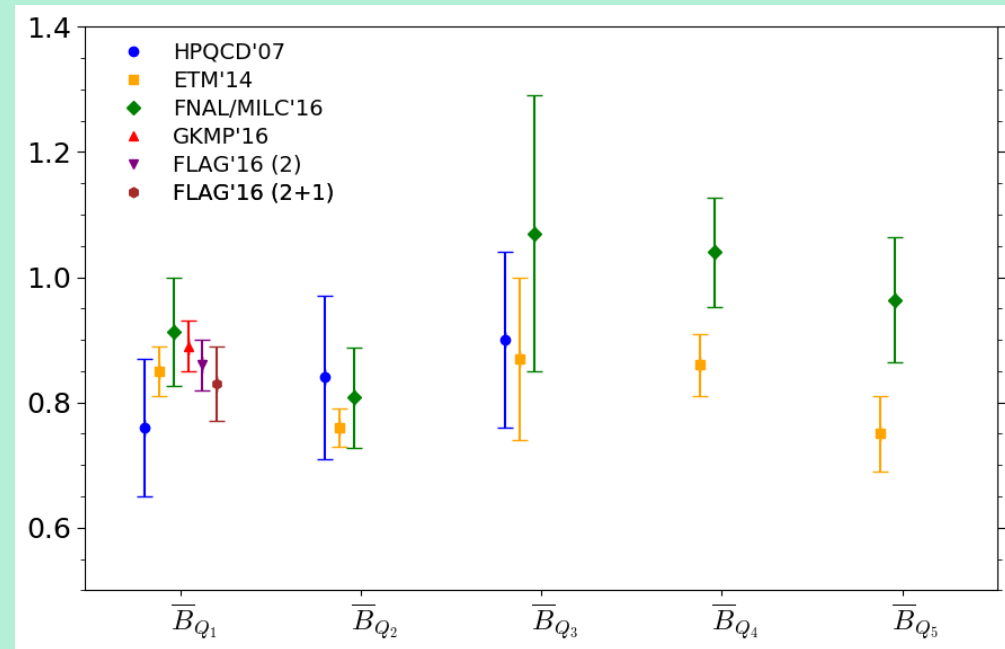
# Status before 2017 – lattice

- $B_s$  Mixing

- Selection of lattice results, all in agreement

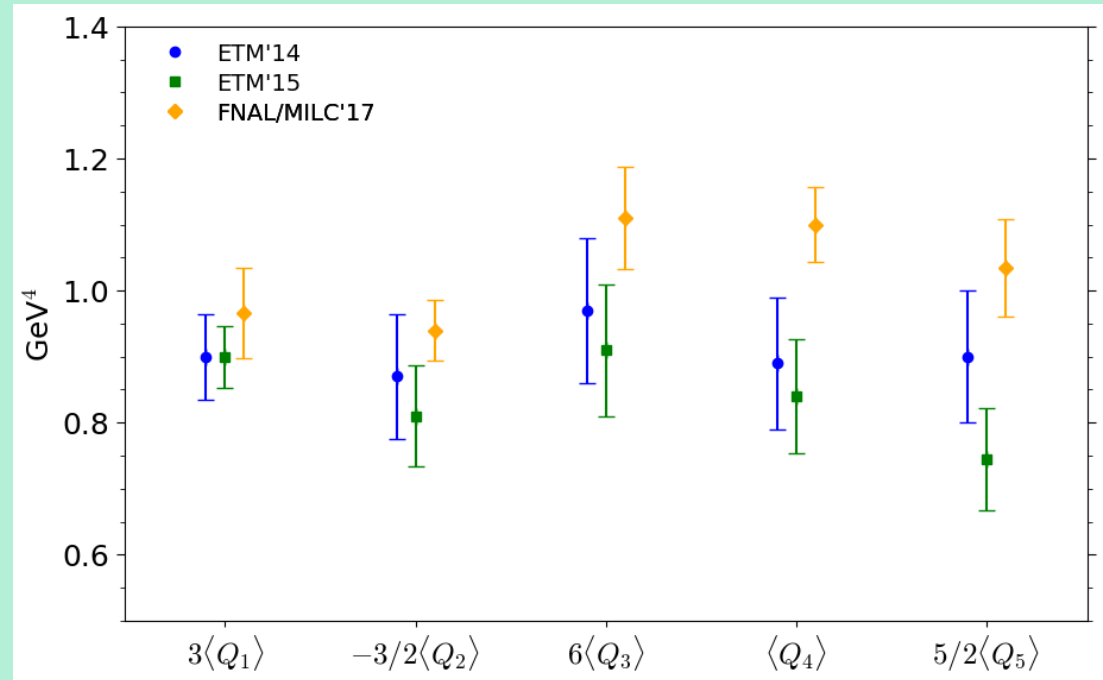
- B Lifetimes

- only old ('98 / '01) lattice results



# Status before 2017 – lattice

- D mixing
  - a handful of lattice results



- D lifetimes



# Status before 2017 – theory

- B Mixing –  $\Delta M_s = 18.3 \pm 2.7 \text{ ps}^{-1}$   
 $\Delta \Gamma_s = 0.088 \pm 0.020 \text{ ps}^{-1}$
- B Lifetimes –  $\tau(B_s)/\tau(B_d) = 1.0005 \pm 0.0011$   
–  $\tau(B^+)/\tau(B_d) = 1.04^{+0.05}_{-0.02}$
- D mixing – ?
- D lifetimes –  $\tau(D^+)/\tau(D^0) = 2.2 \pm 1.7$

# What has happened since?

- New lattice result from Fermilab-MILC included in FLAG average
  - $f_{B_s} \sqrt{B}$  :  $270 \pm 16$  MeV  $\rightarrow$   $274 \pm 8$  MeV
- HQET sum rule calculation
  - Independent determination of non-perturbative matrix elements for all dimension-6 operators
- $V_{cb}$  discrepancy between inclusive / exclusive is perhaps starting to be resolved?
  - (1703.08170, 1707.09509, 1708.07134, talk by Stefan Schacht at LHCb Implications)

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$$\begin{aligned}
 M_3(\omega_1, \omega_2) = & (-2\omega_1)^{3d/2-5} (-2\omega_2)^{3d/2-5} \Gamma^3(d/2 - 1) \\
 & \times \left[ \frac{\Gamma(\frac{3}{2}d - 4) \Gamma^2(5 - \frac{3}{2}d) \Gamma(2 - \frac{d}{2})}{(d - 3)\Gamma(d - 2)} \right. \\
 & + 2 \frac{\Gamma(8 - 3d)}{d - 3} x^{4-3d/2} {}_3F_2 \left( \begin{matrix} 1, d - 2, \frac{3}{2}d - 4 \\ \frac{3}{2}d - 3, 3d - 8 \end{matrix} \middle| \frac{1}{x} \right) \\
 & + \frac{4\pi\Gamma(6 - 2d)x^{3d/2-5}}{(3d - 10)\Gamma(d - 2) \sin(3\pi d)} {}_2F_1 \left( \begin{matrix} 5 - \frac{3}{2}d, 7 - 2d \\ 6 - \frac{3}{2}d \end{matrix} \middle| \frac{1}{x} \right) \\
 & + 2 \frac{\Gamma(8 - 3d)}{d - 3} x^{3d/2-4} {}_3F_2 \left( \begin{matrix} 1, d - 2, \frac{3}{2}d - 4 \\ \frac{3}{2}d - 3, 3d - 8 \end{matrix} \middle| x \right) \\
 & \left. + \frac{4\pi\Gamma(6 - 2d)x^{5-3d/2}}{(3d - 10)\Gamma(d - 2) \sin(3\pi d)} {}_2F_1 \left( \begin{matrix} 5 - \frac{3}{2}d, 7 - 2d \\ 6 - \frac{3}{2}d \end{matrix} \middle| x \right) \right].
 \end{aligned}$$

# HQET sum rules

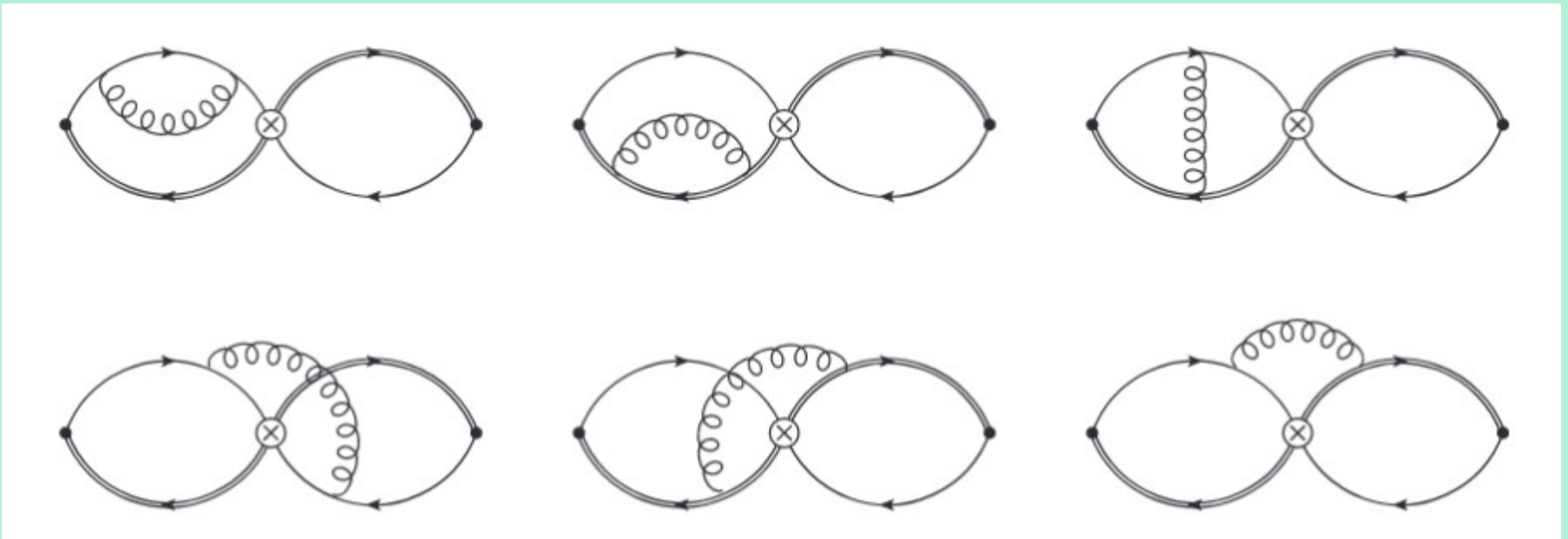
- Made possible by 3-loop calculations done in 2008 by Grozin, Lee (**0812.4522**)
- First steps made by Grozin, Klein, Mannel, Pivovarov in mid 2016 (**1606.06054**)
- Late last year, full set of dim-6 operators done by MK, Lenz, Rauh (**1711.02100**)

# HQET sum rules

- Do all dim 6 operators for mixing AND lifetimes
- How?
  - 3 loop diagrams (with 2 external momenta), reduced using FIRE to those known by Grozin, Lee

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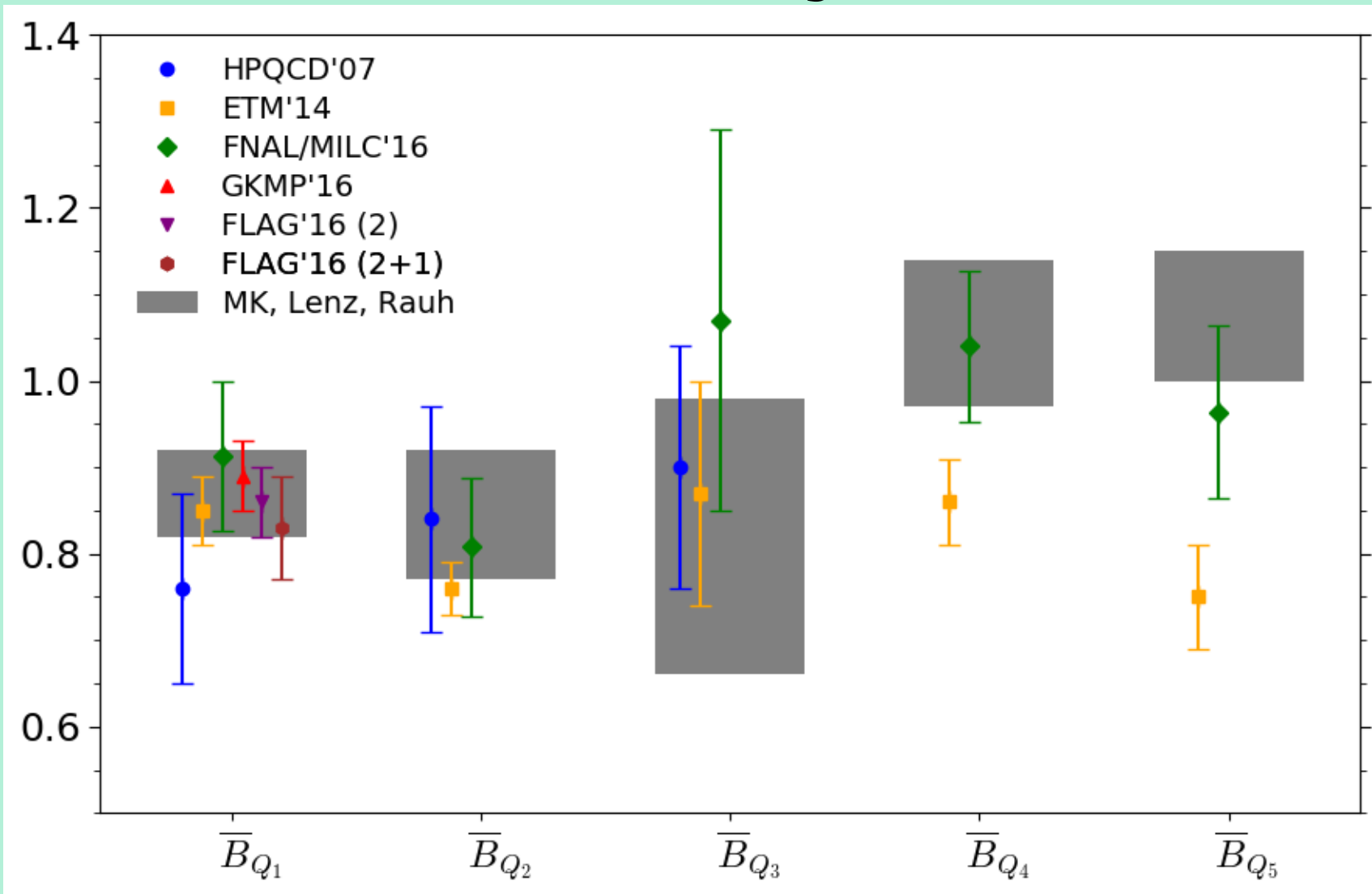
# HQET sum rules

- Do all dim 6 operators for mixing AND lifetimes
- How?
  - 3 loop diagrams (with 2 external momenta), reduced using FIRE to those known by Grozin, Lee
  - HQET running to scale  $m_b$
  - HQET-QCD matching (1-loop) at scale  $m_b$

# HQET sum rules – results

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## B mixing



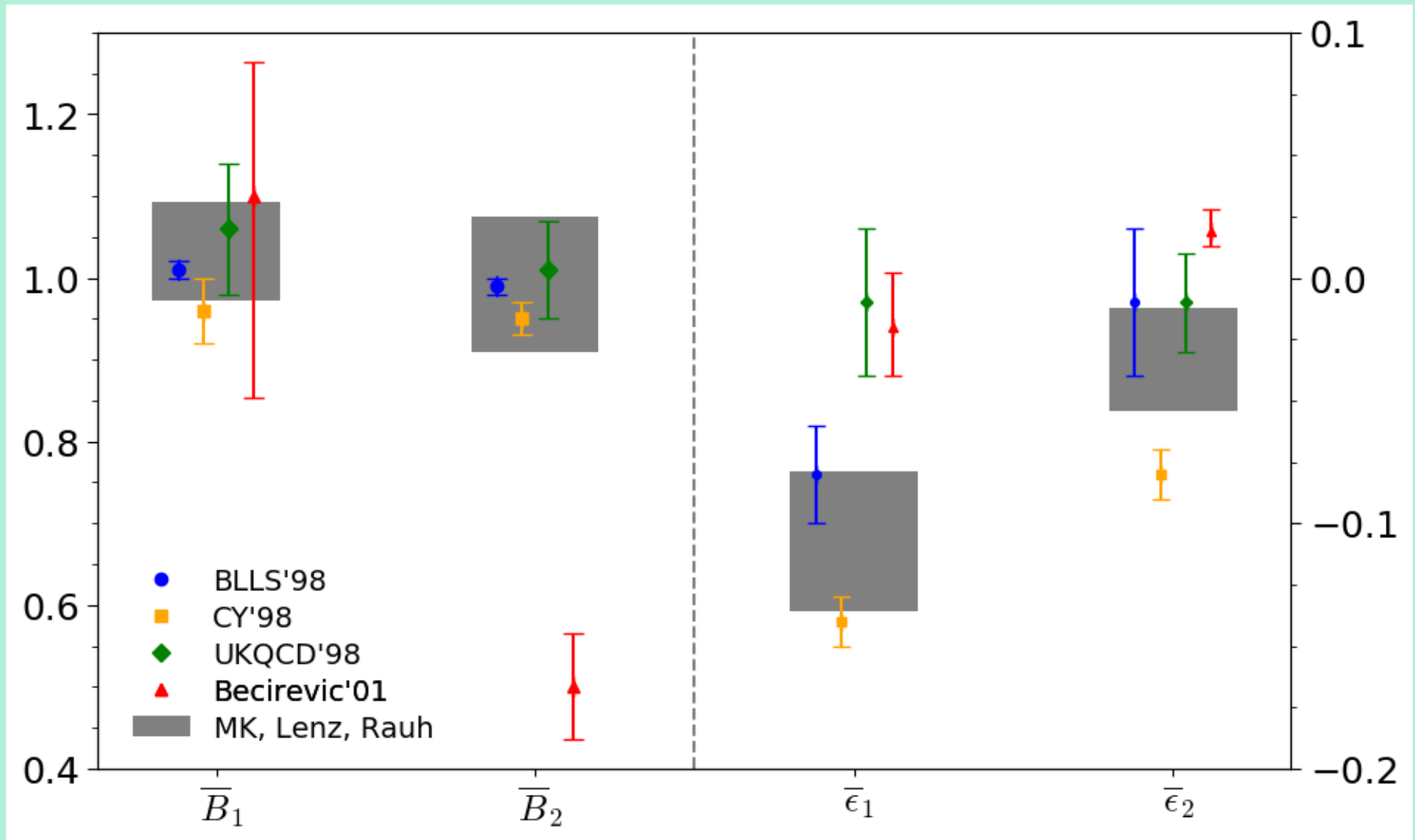
# Effect on observables

- $\Delta M_s = 18.1 \pm 1.9 \text{ ps}^{-1}$
- $\Delta \Gamma_s = 0.079 \pm 0.023 \text{ ps}^{-1}$
- $a_{sl}^s = 2.0 \pm 0.3 \times 10^{-5}$
- Gives errors that are comparable ( $\pm 15\%$ ) with lattice data  $\rightarrow$  lattice not the only game in town



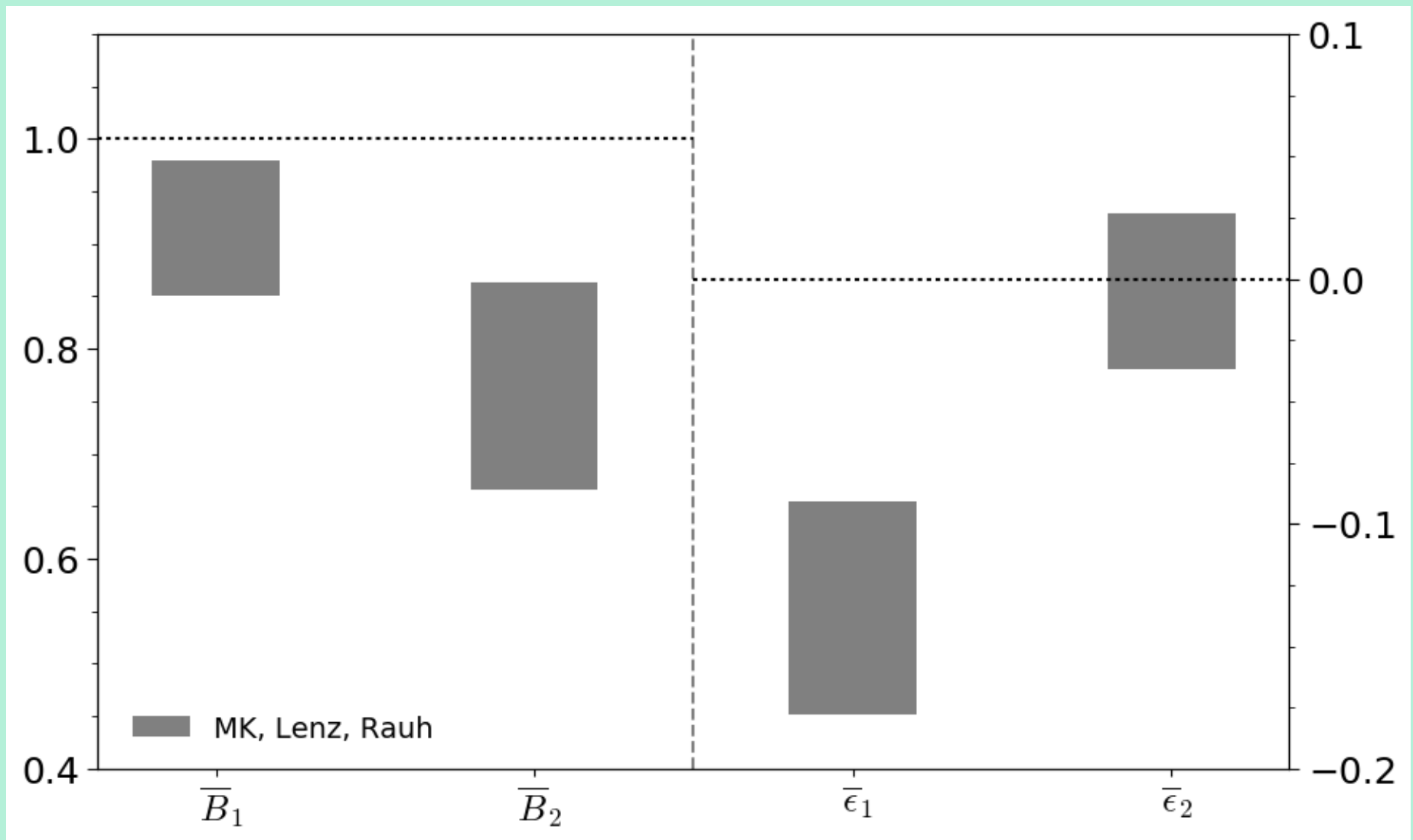
# HQET sum rules – results

B lifetimes



# HQET sum rules – results

D lifetimes



# Effect on observables

- $\tau(B_s)/\tau(B_d) = 0.9994 \pm 0.0025$
- $\tau(B^+)/\tau(B_d) = 1.082^{+0.022}_{-0.026}$
- $\tau(D^+)/\tau(D^0) = 2.7^{+0.7}_{-0.8}$
- For lifetimes, lattice hasn't yet arrived → sum rules the only game in town

# Effects on NP models

- Non-perturbative parameters very important
- Constraints from B mixing depend sensitively on values

# Effects on NP models

- Using the latest FLAG average → much less space for e.g.  $Z'$  model
- See **1712.06572** (Di Luzio, MK, Lenz)

## One constraint to kill them all?

Luca Di Luzio, Matthew Kirk, Alexander Lenz

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### Abstract

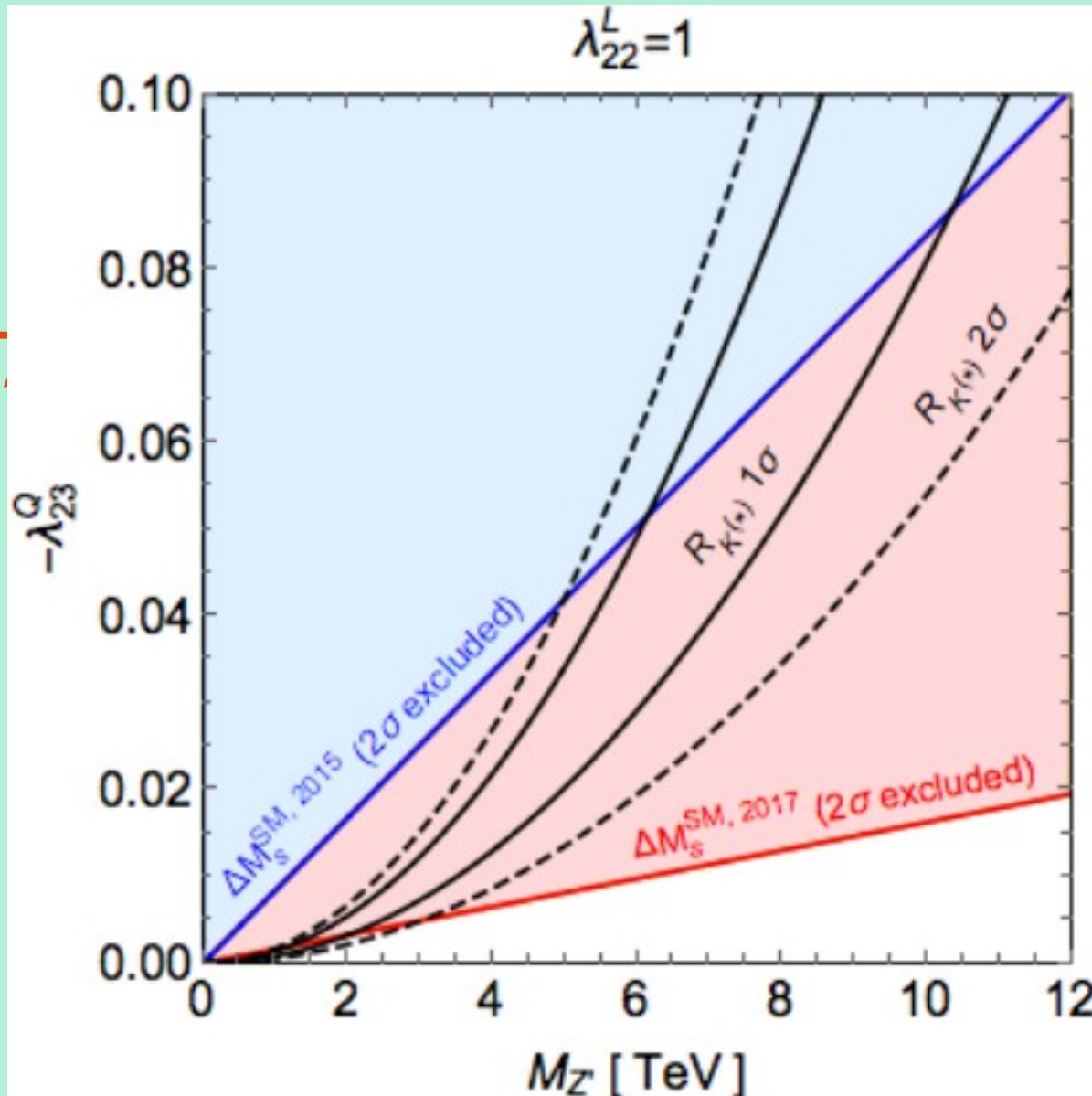
Many new physics models that explain the intriguing anomalies in the  $b$ -quark flavour sector are severely constrained by  $B_s$ -mixing, for which the Standard Model prediction and experiment agreed well until recently. New non-perturbative calculations point, however, in the direction of a small discrepancy in this observable. Using up-to-date inputs to determine  $\Delta M_s^{\text{SM}}$ , we find a severe reduction of the allowed parameter space of  $Z'$  and leptoquark models explaining the  $B$ -anomalies. Remarkably, in the former case the upper bound on the  $Z'$  mass approaches dangerously close to the energy scales already probed by the LHC. We finally identify some model building directions in order to alleviate the tension with  $B_s$ -mixing.

*Keywords:* New Physics, B-Physics, B-mixing

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# Effects on NP models

- Using space
- See 17



less

# Effects on NP models

- Good example of why independent determinations necessary
- From different lattice groups AND other methods

# What next?

- Determination of dimension-7 operators

$$R_2 = \frac{1}{m_b^2} \bar{b}_i \overleftarrow{D}_\lambda \gamma_\mu (1 - \gamma^5) D^\lambda q_i \bar{b}_j \gamma^\mu (1 - \gamma^5) q_j$$

from lattice / sum rules – reduce error in  $\Delta \Gamma_s$

- Lattice confirmation of dimension-6



# What next?

- Determination of dimension-7 operators

$$R_2 = \frac{1}{m_b^2} \bar{b}_i \overleftarrow{D}_\lambda \gamma_\mu (1 - \gamma^5) D^\lambda q_i \bar{b}_j \gamma^\mu (1 - \gamma^5) q_j$$

from lattice / sum rules – reduce error in  $\Delta \Gamma_s$

- Lattice confirmation of dimension-6
- HPQCD working on both
- MK, Rauh, Lenz working on dim-7 now

# What next?

- Determination of dimension-7 operators

$$R_2 = \frac{1}{m_b^2} \bar{b}_i \overleftarrow{D}_\lambda \gamma_\mu (1 - \gamma^5) D^\lambda q_i \bar{b}_j \gamma^\mu (1 - \gamma^5) q_j$$

from lattice / sum rules – reduce error in  $\Delta \Gamma_s$

- Lattice confirmation of dimension-6
- Know  $\tau(B_s)/\tau(B_d)$  better from experiment – while already doing very well, theory is currently ahead

Thanks!

Backup

# Mixing operators

$$Q_1 = \bar{b}_i \gamma_\mu (1 - \gamma^5) q_i \bar{b}_j \gamma^\mu (1 - \gamma^5) q_j,$$

$$Q_2 = \bar{b}_i (1 - \gamma^5) q_i \bar{b}_j (1 - \gamma^5) q_j,$$

$$Q_4 = \bar{b}_i (1 - \gamma^5) q_i \bar{b}_j (1 + \gamma^5) q_j,$$

$$Q_3 = \bar{b}_i (1 - \gamma^5) q_j \bar{b}_j (1 - \gamma^5) q_i,$$

$$Q_5 = \bar{b}_i (1 - \gamma^5) q_j \bar{b}_j (1 + \gamma^5) q_i,$$

# Lifetime operators

$$Q_1^q = \bar{b}\gamma_\mu(1 - \gamma^5)q \bar{q}\gamma^\mu(1 - \gamma^5)b,$$

$$Q_2^q = \bar{b}(1 - \gamma^5)q \bar{q}(1 + \gamma^5)b,$$

$$T_1^q = \bar{b}\gamma_\mu(1 - \gamma^5)T^A q \bar{q}\gamma^\mu(1 - \gamma^5)T^A b,$$

$$T_2^q = \bar{b}(1 - \gamma^5)T^A q \bar{q}(1 + \gamma^5)T^A b.$$

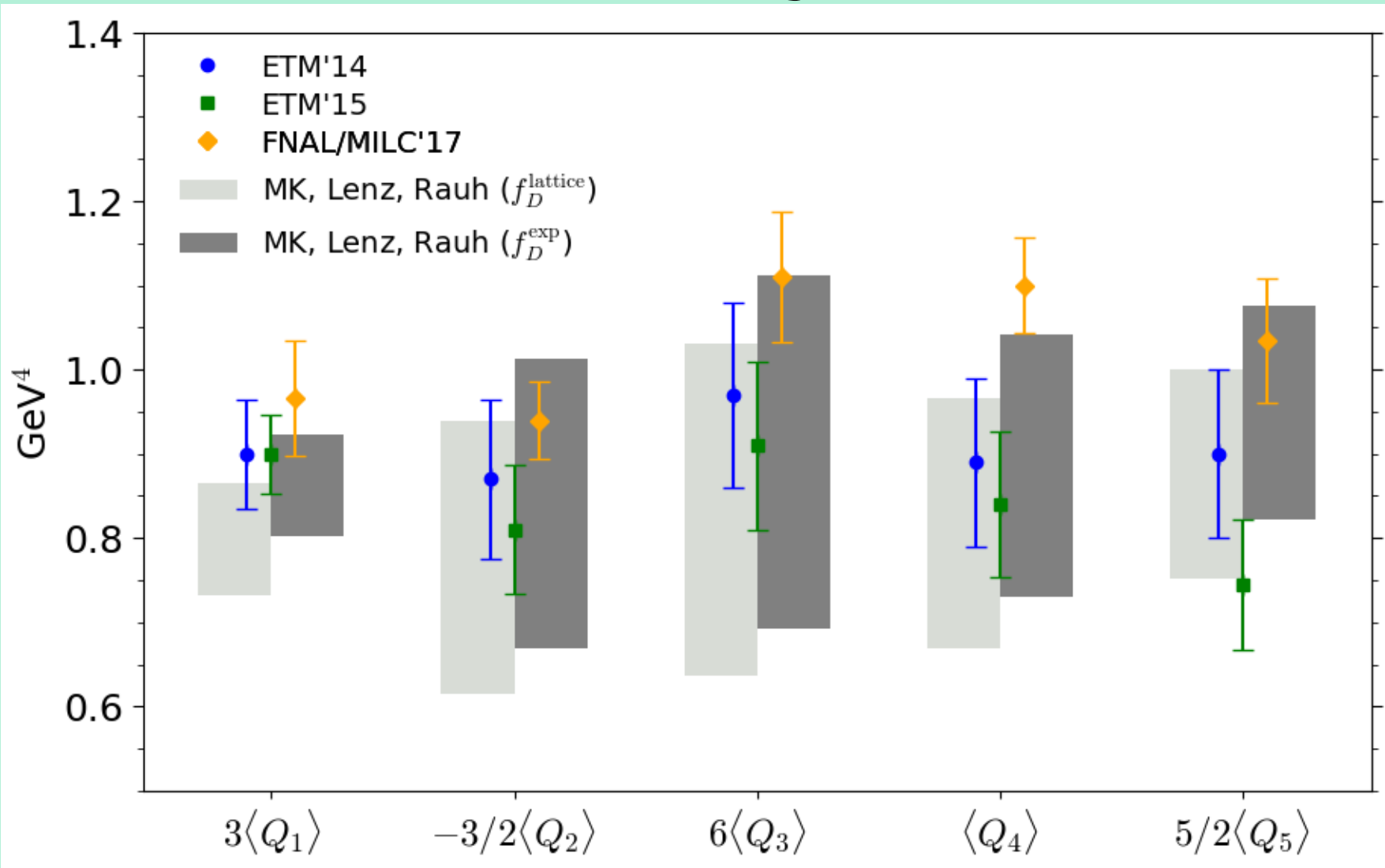
# Effects on NP models

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Source	$f_{B_s} \sqrt{\hat{B}}$	$\Delta M_s^{\text{SM}}$
HPQCD14 [116]	$(247 \pm 12) \text{ MeV}$	$(16.2 \pm 1.7) \text{ ps}^{-1}$
HQET-SR [71]	$(261 \pm 8) \text{ MeV}$	$(18.1 \pm 1.1) \text{ ps}^{-1}$
ETMC13 [117]	$(262 \pm 10) \text{ MeV}$	$(18.3 \pm 1.5) \text{ ps}^{-1}$
HPQCD09 [118] = FLAG13 [119]	$(266 \pm 18) \text{ MeV}$	$(18.9 \pm 2.6) \text{ ps}^{-1}$
<b>FLAG17 [65]</b>	<b><math>(274 \pm 8) \text{ MeV}</math></b>	<b><math>(20.01 \pm 1.25) \text{ ps}^{-1}</math></b>
Fermilab16 [67]	$(274.6 \pm 4) \text{ MeV}$	$(20.1 \pm 0.7) \text{ ps}^{-1}$
HPQCD06 [120]	$(281 \pm 20) \text{ MeV}$	$(21.0 \pm 3.0) \text{ ps}^{-1}$
RBC/UKQCD14 [121]	$(290 \pm 20) \text{ MeV}$	$(22.4 \pm 3.4) \text{ ps}^{-1}$
Fermilab11 [122]	$(291 \pm 18) \text{ MeV}$	$(22.6 \pm 2.8) \text{ ps}^{-1}$

# HQET sum rules – results

## D mixing





# SINCE YEARS OF BEGGING DID NOT HELP – IT'S TIME TO PROVOKE

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*Lifetimes are too heavy for lattice physicists!*

**The strongest lattice researcher alive**



**Arbitrary sum rule researcher**



Matrix elements for lifetimes of HEAVY mesons