Meson mixing: bag parameters and sum rules

Matthew Kirk

based on **1711.02100 (MK, Rauh, Lenz)**

1712.06572 (Di Luzio, MK, Lenz)

Experimental status (~2017)

- B_s Mixing
  - ΔM_s is extremely well measured (0.1% uncertainty)
  - $\Delta \Gamma_s$ known with sub 10% uncertainty
- B lifetime ratios
  - $\tau(B_s)/\tau(B_d)$ known with < 0.25% uncertainty
- D Mixing
  - First > 5 sigma measurement from LHCb in 2012
  - $O(10\,\%)$ accuracy
- D lifetime ratios
  - $\tau(D^+)/\tau(D^0)$ known with <1% uncertainty

Lattice status (~2017)

- Lattice can determine non-perturbative parameters – essentially they do the path integral numerically
- We are interested in overlap between meson/anti-meson states

$$\langle \text{meson} | \text{four quark operator} | \text{anti-meson} \rangle \sim B$$

or

$$\langle \text{meson} | \text{four quark operator} | \text{meson} \rangle \sim B/\epsilon$$

Vacuum saturation approximation

$$\langle B_s|(\bar{q}\,\Gamma\,b)(\bar{q}\,\Gamma\,b)|\overline{B_s}\rangle = \sum_{\text{all states}} \langle B_s|(\bar{q}\,\Gamma\,b)|X\rangle\langle X|(\bar{q}\,\Gamma\,b)|\overline{B_s}\rangle$$

$$\approx \langle B_s|(\bar{q}\,\Gamma\,b)|0\rangle\langle 0|(\bar{q}\,\Gamma\,b)|\overline{B_s}\rangle$$

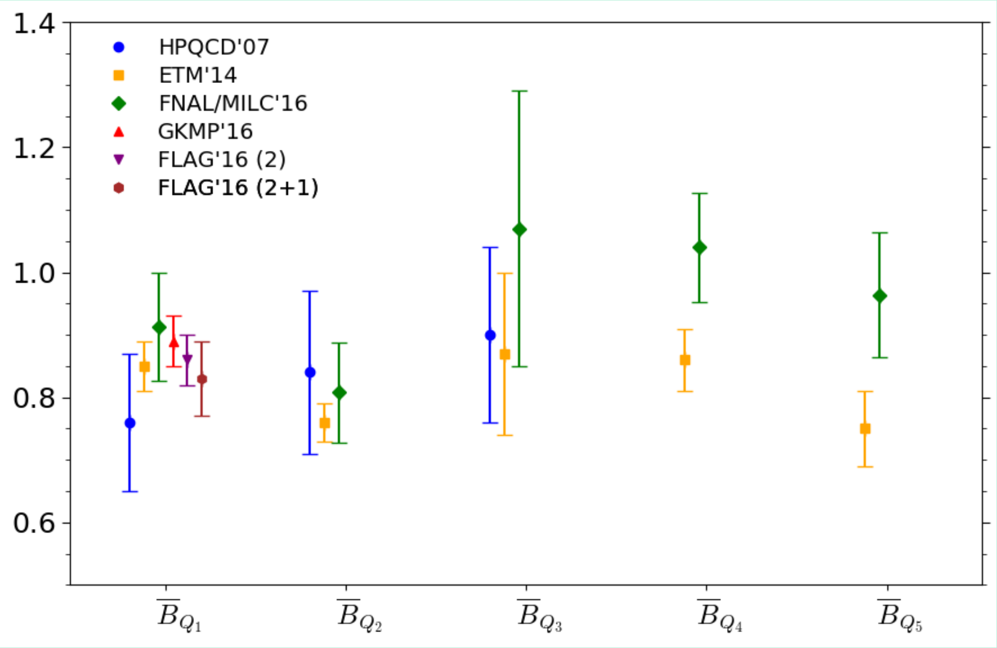
These then look like decay constants for meson to vacuum – extracted from experimental decay width

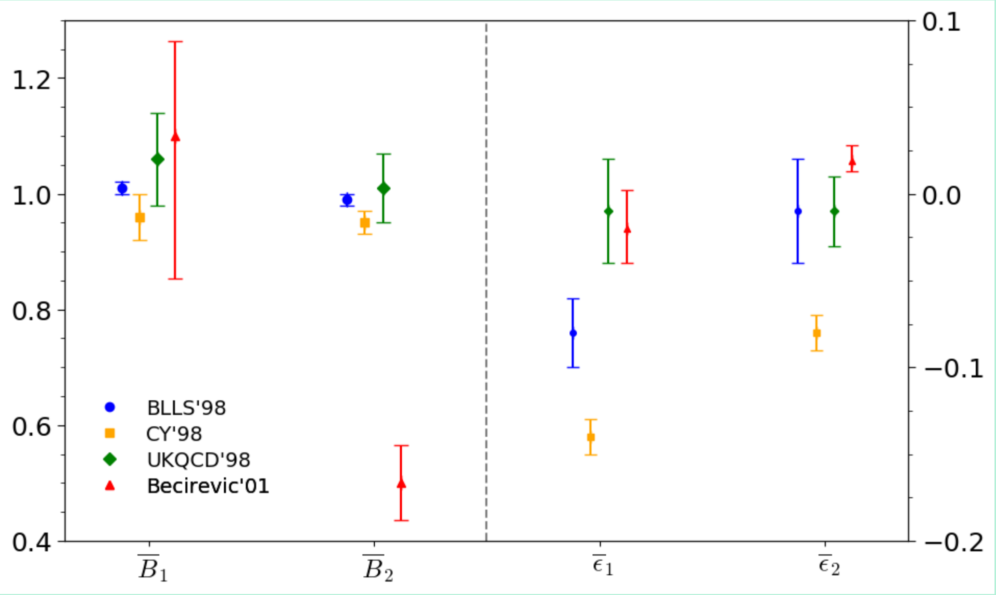
$$\langle B_s|(\bar{q}\,\Gamma\,b)(\bar{q}\,\Gamma\,b)|\overline{B_s}\rangle = B_\Gamma \langle B_s|(\bar{q}\,\Gamma\,b)|0\rangle\langle 0|(\bar{q}\,\Gamma\,b)|\overline{B_s}\rangle$$

Bag parameter

Lattice status (~2017)

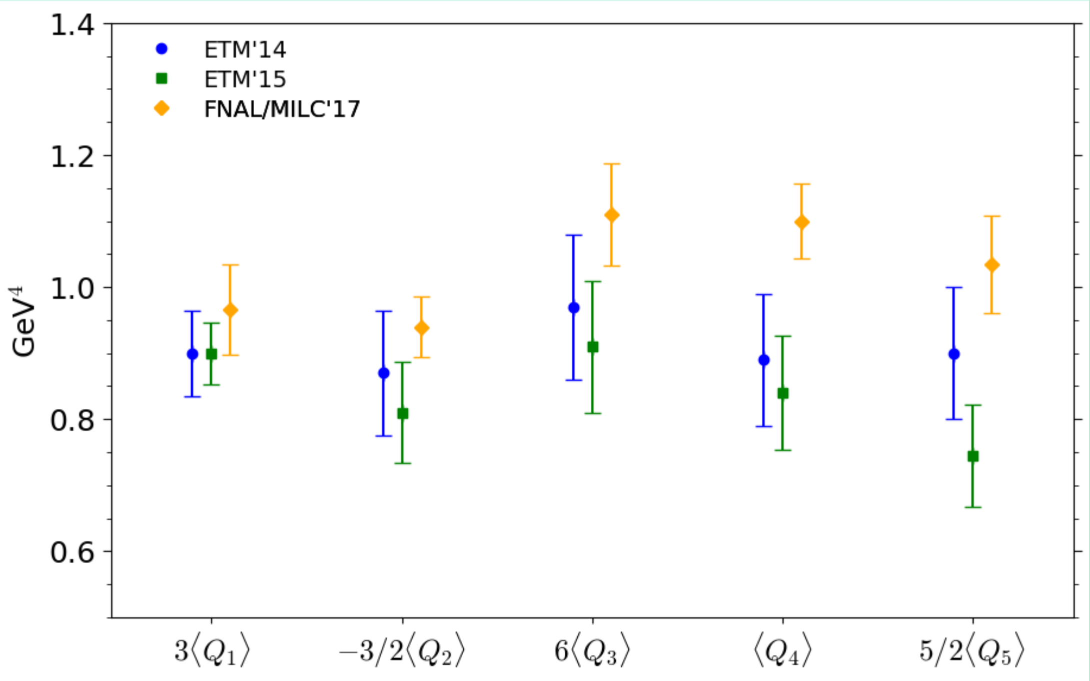
- B_s Mixing
  - Selection of lattice results, all in agreement
- B Lifetimes
  - only old ('98 / '01) lattice results





Lattice status (~2017)

- D mixing
  - a handful of lattice results



- D lifetimes

Theory status (~2015)

- B Mixing – $\Delta M_s = 18.3 \pm 2.7\,\mathrm{ps}^{-1}$

  $\Delta \Gamma_s = 0.088 \pm 0.020\,\mathrm{ps}^{-1}$
- B Lifetimes – $\tau(B_s)/\tau(B_d) = 1.0005 \pm 0.0011$

  – $\tau(B^+)/\tau(B_d) = 1.04^{+0.05}_{-0.02}$
- D mixing – ?
- D lifetimes – $\tau(D^+)/\tau(D^0) = 2.2 \pm 1.7$

What has happened since?

- New lattice result from Fermilab-MILC included in FLAG average
  - $f_{B_s}\sqrt{B}$: 270 ± 16 MeV → 274 ± 8 MeV
- HQET sum rule calculation
  - Independent determination of non-perturbative matrix elements for all dimension-6 operators
- V_{cb} discrepancy between inclusive / exclusive is perhaps starting to be resolved?
  - (**1703.08170**, **1707.09509**, **1708.07134**, talk by **Stefan Schacht** at **LHCb Implications**)

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Intro to Effective Field Theories

EFTs In a Nutshell

- Applicable in any theory with large scale separation
- Often assumption that "heavy" particle mediates an interaction which is approximated to be point-like
- Create vertices not seen in the SM, with **Wilson Coefficients** behaving as effective couplings
- Calculations can be performed with a precision up to the ~ ratio of the two scales

Top-Down Vs Bottom-Up

Top-Down

- Start with full UV-complete theory
- Integrate out heavy fields
- Generate mathematically simpler theory
- Wilson coefficients defined by variables of full theory

$$G_F = \frac{\sqrt{2}g^2}{8M_W^2}$$

Suppressed by "heavy" scale

Bottom-Up

- Build basis of operators without making any connection to a UV complete theory
- Wilson coefficients entirely unspecified

Intro to Heavy Quark Effective Theory

- In mesons containing *b* quarks, we have two scales:
  - the *b* quark mass (~ 4 GeV)
  - the light quark masses / the QCD scale (~ 100 meV)
- So we can take an effective theory approach

Intro to Heavy Quark Effective Theory

- Intuitive picture – think of *b* quark as being at rest, and the light quarks / gluons moving around in the QCD potential it creates
- Analogous to way of studying hydrogen atom – proton at rest and electron moving around in the QED (i.e. electromagnetic) potential it creates

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Introduction to sum rules

- Consider two point function:

$$\Pi_{\mu\nu}(q) = i \int d^4x \, e^{iq\cdot x} \langle 0 | \, T\{j_\mu(x) j_\nu(0)\} \, | 0 \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi(q^2)$$

- It is analytic in q, with poles at bound states and branch cut for continuum of excited states
- At large |q|, everything off-shell, high energy → perturbation theory works
- Relate the two regions by Cauchy's theorem

Introduction to sum rules

$$\Pi_b(Q^2) = \frac{1}{2\pi i} \oint_C dz \frac{\Pi_b(z)}{z - Q^2}$$

$$\Pi_b(Q^2) = \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\mathrm{Im}\,\Pi_b(s)}{s - Q^2} + \frac{1}{2\pi i} \oint_{\bigcirc} dz \frac{\Pi_b(z)}{z - Q^2}.$$

Introduction to sum rules

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Determine in perturbation theory

Proportional to the cross-section for quark pair production – get from experiment

Remove by taking derivatives, Borel transformation, ...

HQET sum rules

- Made possible by 3-loop calculations done in 2008 by Grozin, Lee (0812.4522)

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$$
\begin{aligned}
M_3(\omega_1,\omega_2) = {} & (-2\omega_1)^{3d/2-5}(-2\omega_2)^{3d/2-5}\Gamma^3(d/2-1) \\
& \times \left[\frac{\Gamma\left(\frac{3}{2}d-4\right)\Gamma^2\left(5-\frac{3}{2}d\right)\Gamma\left(2-\frac{d}{2}\right)}{(d-3)\Gamma(d-2)} \right. \\
& \quad + 2\frac{\Gamma(8-3d)}{d-3}x^{4-3d/2}\,{}_3F_2\left(\left.\begin{matrix} 1, d-2, \frac{3}{2}d-4 \\ \frac{3}{2}d-3, 3d-8 \end{matrix}\,\right|\,\frac{1}{x}\right) \\
& \quad + \frac{4\pi\Gamma(6-2d)x^{3d/2-5}}{(3d-10)\Gamma(d-2)\sin(3\pi d)}\,{}_2F_1\left(\left.\begin{matrix} 5-\frac{3}{2}d, 7-2d \\ 6-\frac{3}{2}d \end{matrix}\,\right|\,\frac{1}{x}\right) \\
& \quad + 2\frac{\Gamma(8-3d)}{d-3}x^{3d/2-4}\,{}_3F_2\left(\left.\begin{matrix} 1, d-2, \frac{3}{2}d-4 \\ \frac{3}{2}d-3, 3d-8 \end{matrix}\,\right|\,x\right) \\
& \quad \left. + \frac{4\pi\Gamma(6-2d)x^{5-3d/2}}{(3d-10)\Gamma(d-2)\sin(3\pi d)}\,{}_2F_1\left(\left.\begin{matrix} 5-\frac{3}{2}d, 7-2d \\ 6-\frac{3}{2}d \end{matrix}\,\right|\,x\right)\right].
\end{aligned}
$$

HQET sum rules

- Made possible by 3-loop calculations done in 2008 by Grozin, Lee (0812.4522)
- First steps made by Grozin, Klein, Mannel, Pivovarov in mid 2016 (1606.06054)
- Late last year, full set of dim-6 operators done by MK, Lenz, Rauh (1711.02100)

HQET sum rules

- Do all dim 6 operators for mixing AND lifetimes
- How?
  - 3 loop diagrams (with 2 external momenta), reduced using FIRE to those known by Grozin, Lee
  - HQET running to scale m_b
  - HQET-QCD matching (1-loop) at scale m_b

HQET sum rules

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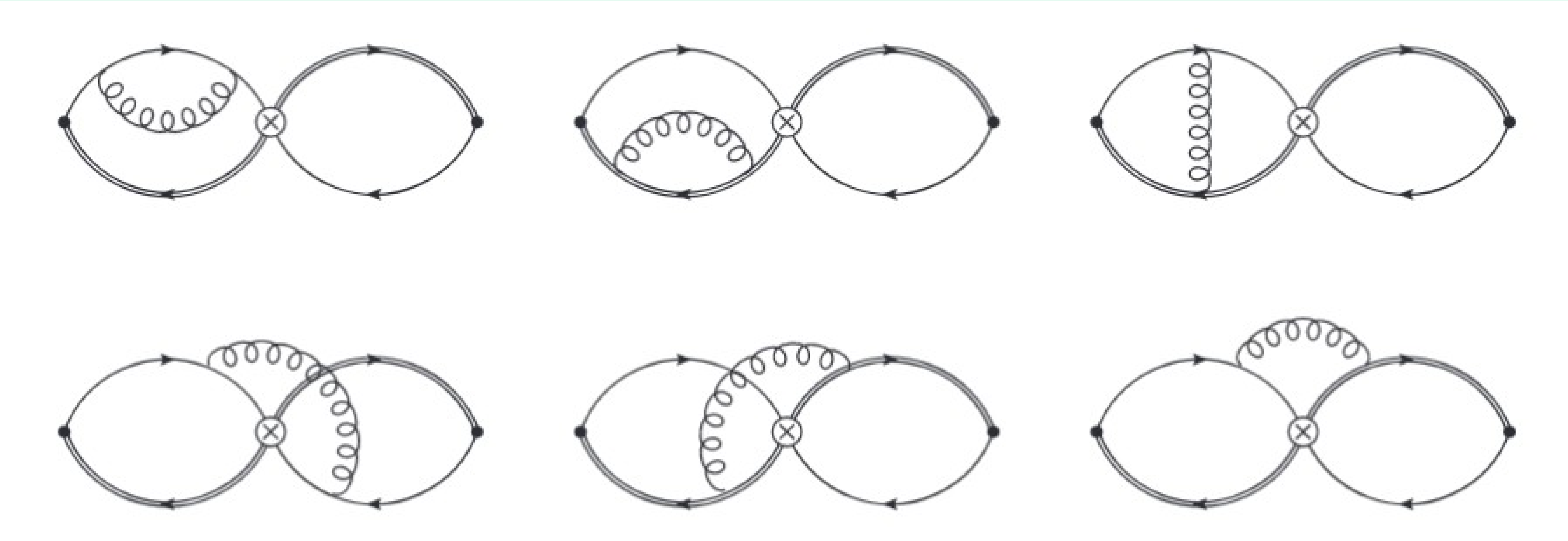

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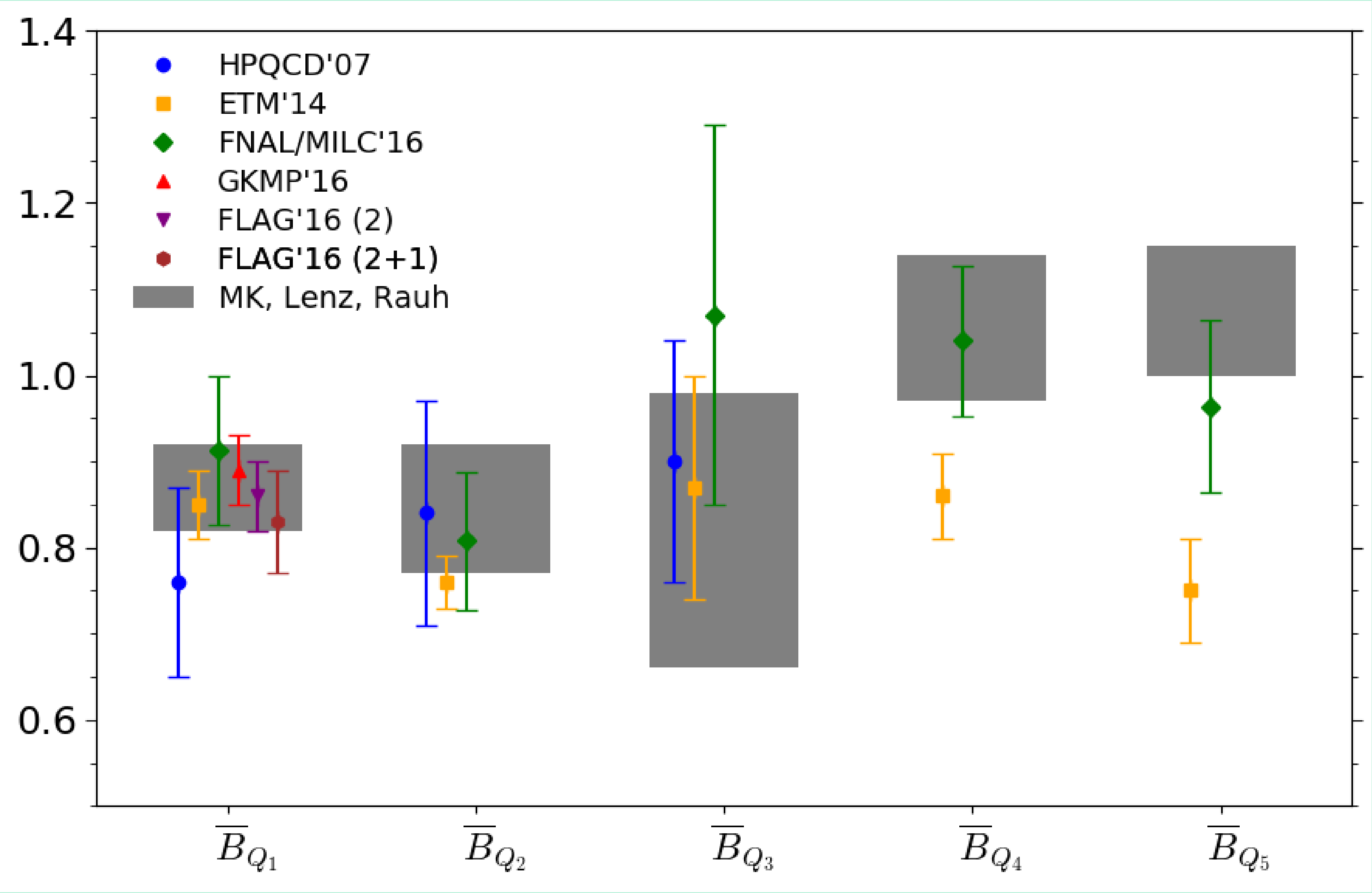
HQET sum rules – results

HQET sum rules – results

B mixing



Effect on observables

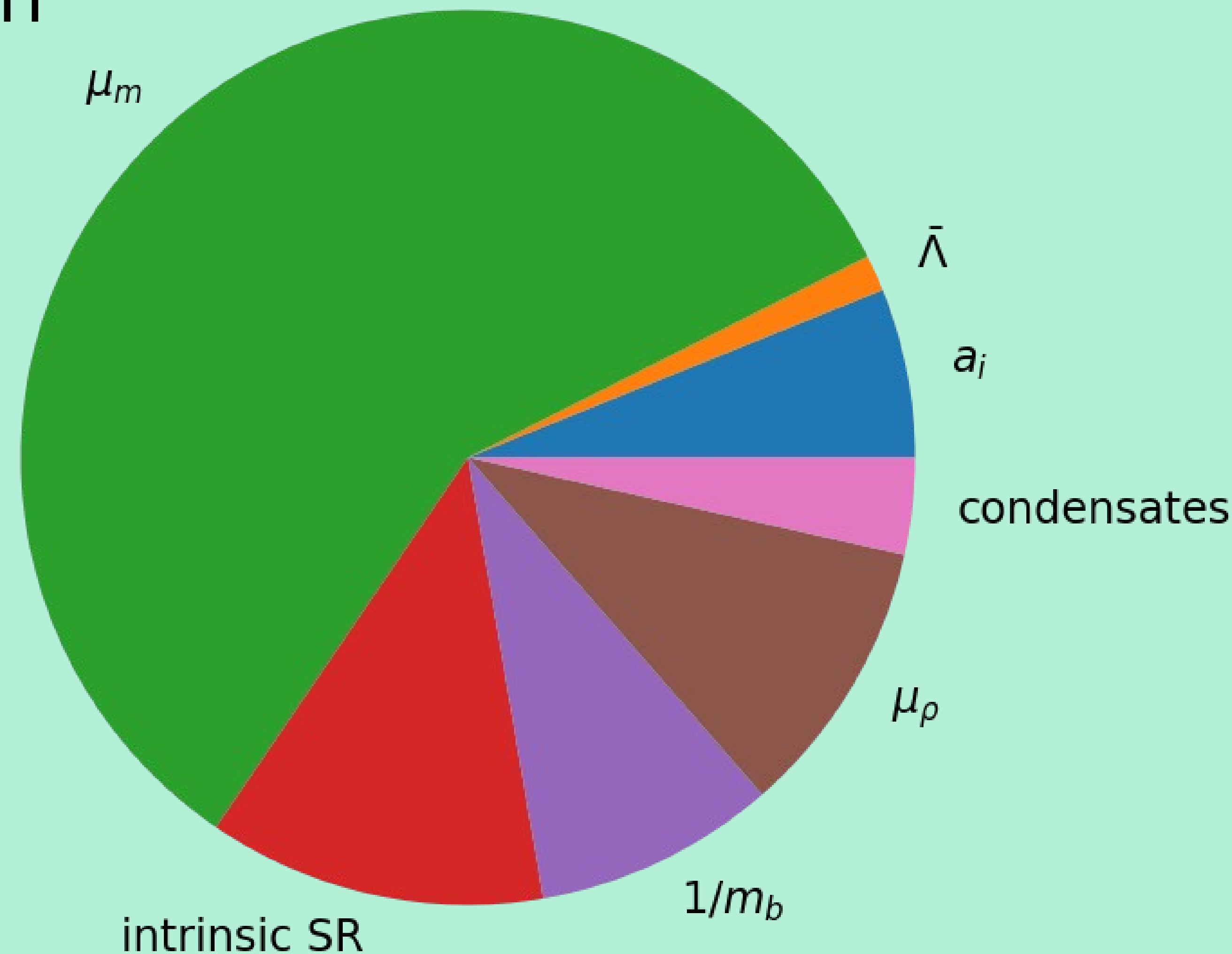
- $\Delta M_s = 18.1 \pm 1.9\,\mathrm{ps}^{-1}$
- $\Delta \Gamma_s = 0.079 \pm 0.023\,\mathrm{ps}^{-1}$
- $a_{sl}^s = 2.0 \pm 0.3 \times 10^{-5}$

- Gives errors that are comparable ($\pm 15\,\%$) with lattice data → lattice not the only game in town

Note on make-up of errors

- Our calculation gives a total uncertainty of ~5-10%

- Dominant uncertainty is the matching



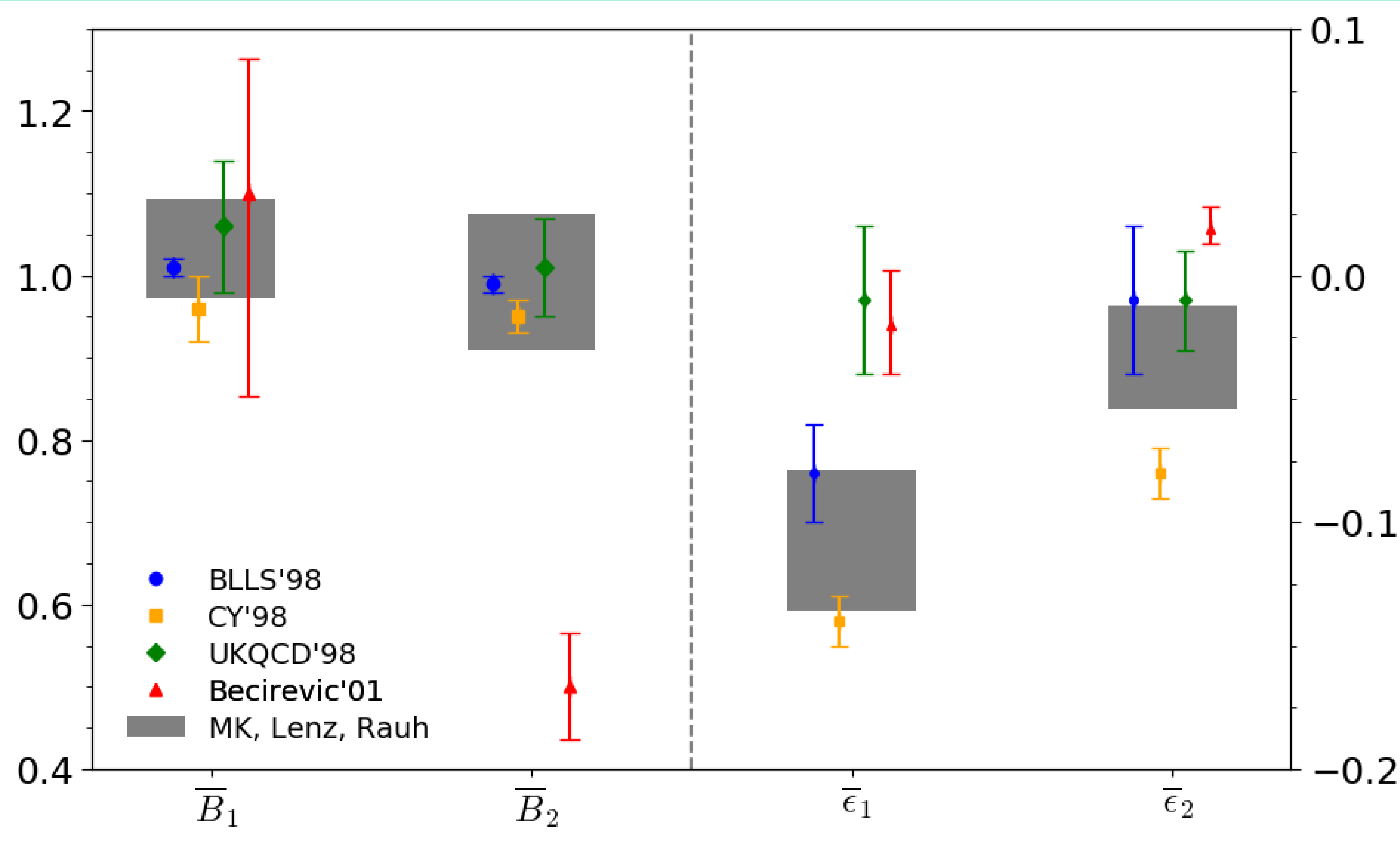
Note on make-up of errors

- Nice trick is that we can calculate the deviation of bag parameters from 1
- Allows our errors to be much smaller than you might expect

- Define $\Delta B \equiv B - 1$
- $\Delta B_{\tilde{Q}} = -0.09 \pm 0.03$
- $B_{\tilde{Q}} = 0.91 \pm 0.03$

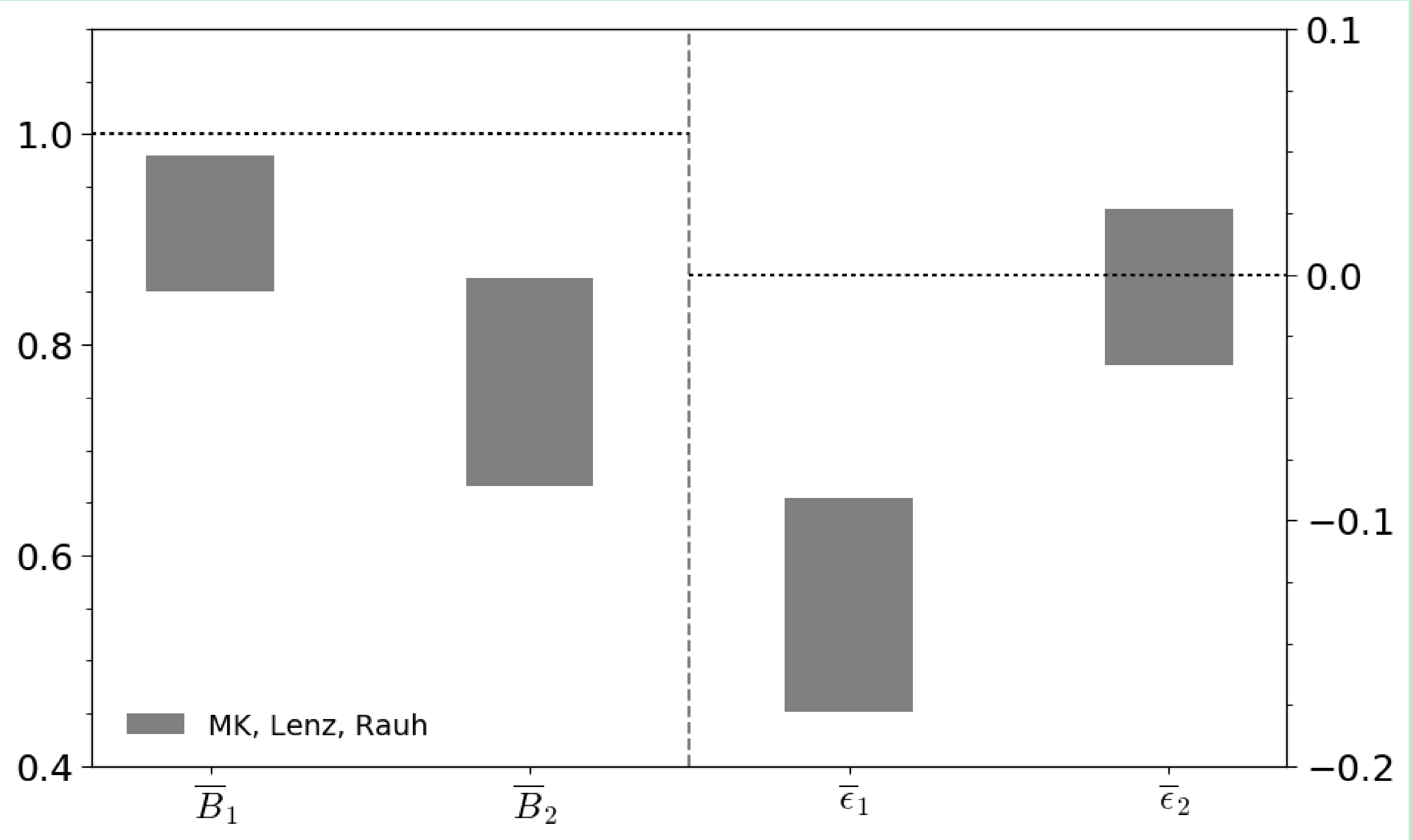
HQET sum rules – results

B lifetimes



HQET sum rules – results

D lifetimes



Effect on observables

- $\tau(B_s)/\tau(B_d) = 0.9994 \pm 0.0025$
- $\tau(B^+)/\tau(B_d) = 1.082^{+0.022}_{-0.026}$
- $\tau(D^+)/\tau(D^0) = 2.7^{+0.7}_{-0.8}$

- For lifetimes, lattice hasn't yet arrived → sum rules the only game in town

Effects on NP models

- Non-perturbative parameters very important
- Constraints from B mixing depend sensitively on values

Source	$f_{B_s}\sqrt{\hat{B}}$	ΔM_s^{SM}
HPQCD14 [116]	(247 ± 12) MeV	$(16.2 \pm 1.7)\,\mathrm{ps}^{-1}$
HQET-SR [71]	(261 ± 8) MeV	$(18.1 \pm 1.1)\,\mathrm{ps}^{-1}$
ETMC13 [117]	(262 ± 10) MeV	$(18.3 \pm 1.5)\,\mathrm{ps}^{-1}$
HPQCD09 [118] = FLAG13 [119]	(266 ± 18) MeV	$(18.9 \pm 2.6)\,\mathrm{ps}^{-1}$
FLAG17 [65]	$\mathbf{(274 \pm 8)}$ **MeV**	$\mathbf{(20.01 \pm 1.25)\,ps^{-1}}$
Fermilab16 [67]	(274.6 ± 4) MeV	$(20.1 \pm 0.7)\,\mathrm{ps}^{-1}$
HPQCD06 [120]	(281 ± 20) MeV	$(21.0 \pm 3.0)\,\mathrm{ps}^{-1}$
RBC/UKQCD14 [121]	(290 ± 20) MeV	$(22.4 \pm 3.4)\,\mathrm{ps}^{-1}$
Fermilab11 [122]	(291 ± 18) MeV	$(22.6 \pm 2.8)\,\mathrm{ps}^{-1}$

Effects on NP models

- Using the latest FLAG average → much less space for e.g. Z' model
- See **1712.06572** (Di Luzio, MK, Lenz)

One constraint to kill them all?

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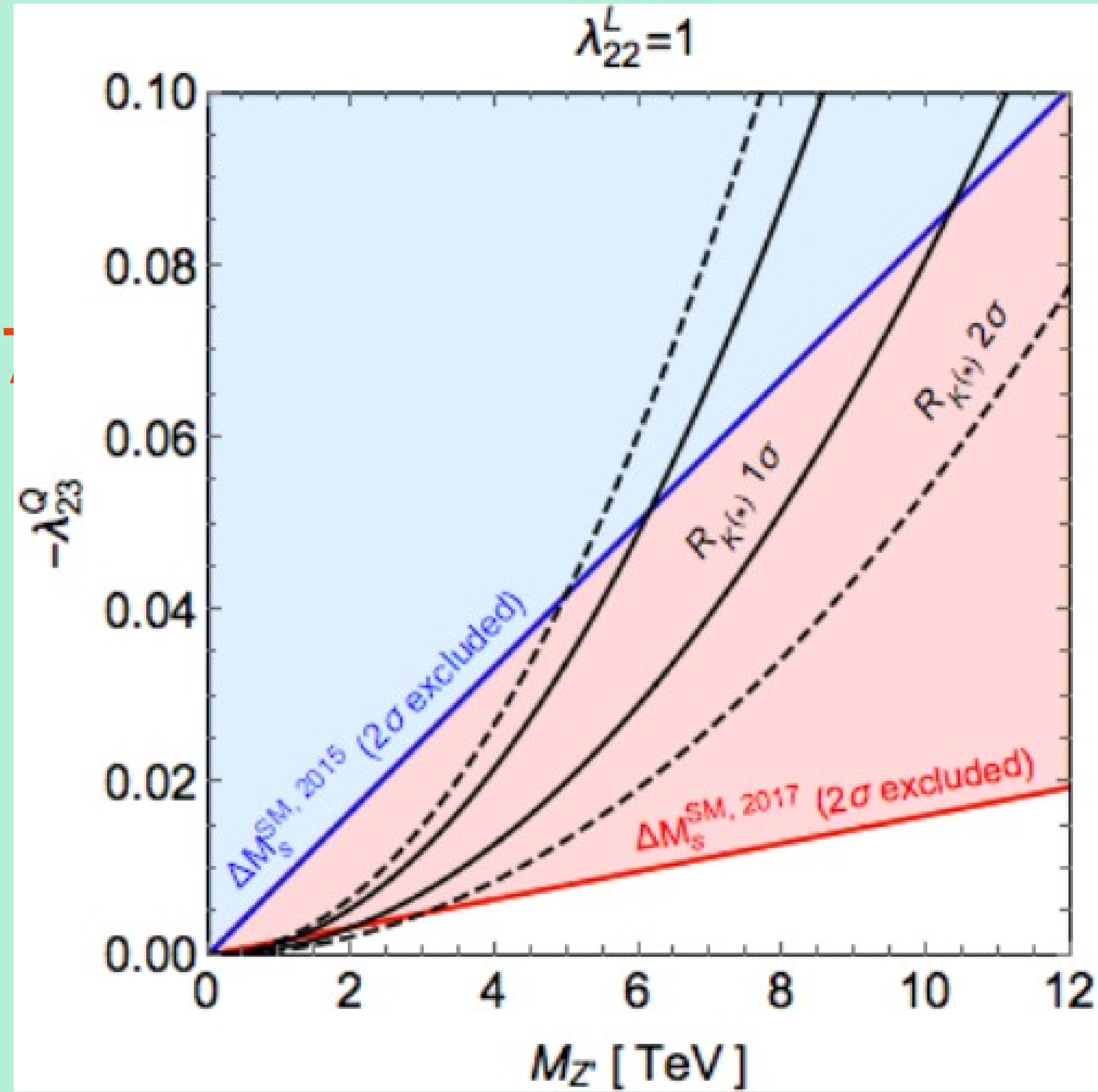
Abstract

Many new physics models that explain the intriguing anomalies in the b-quark flavour sector are severely constrained by B_s-mixing, for which the Standard Model prediction and experiment agreed well until recently. New non-perturbative calculations point, however, in the direction of a small discrepancy in this observable. Using up-to-date inputs to determine ΔM_s^{SM}, we find a severe reduction of the allowed parameter space of Z' and leptoquark models explaining the B-anomalies. Remarkably, in the former case the upper bound on the Z' mass approaches dangerously close to the energy scales already probed by the LHC. We finally identify some model building directions in order to alleviate the tension with B_s-mixing.

Keywords: New Physics, B-Physics, B-mixing

Effects on NP models

- Using … n less
  space …
- See 17…



Effects on NP models

- Good example of why independent determinations necessary
- From different lattice groups AND other methods

What next?

- Determination of dimension-7 operators

$$R_2 = \frac{1}{m_b^2} \bar{b}_i \overleftarrow{D}_\lambda \gamma_\mu (1-\gamma^5) D^\lambda q_i \; \bar{b}_j \gamma^\mu (1-\gamma^5) q_j$$

  from lattice / sum rules – reduce error in $\Delta\Gamma_s$

- Lattice confirmation of dimension-6

What next?

- Determination of dimension-7 operators

$$R_2 = \frac{1}{m_b^2} \bar{b}_i \overleftarrow{D}_\lambda \gamma_\mu (1-\gamma^5) D^\lambda q_i \; \bar{b}_j \gamma^\mu (1-\gamma^5) q_j$$

  from lattice / sum rules – reduce error in $\Delta\Gamma_s$

- Lattice confirmation of dimension-6

- HPQCD working on both
- MK, Rauh, Lenz working on dim-7 now

What next?

- Determination of dimension-7 operators

$$R_2 = \frac{1}{m_b^2} \bar{b}_i \overleftarrow{D}_\lambda \gamma_\mu (1-\gamma^5) D^\lambda q_i \; \bar{b}_j \gamma^\mu (1-\gamma^5) q_j$$

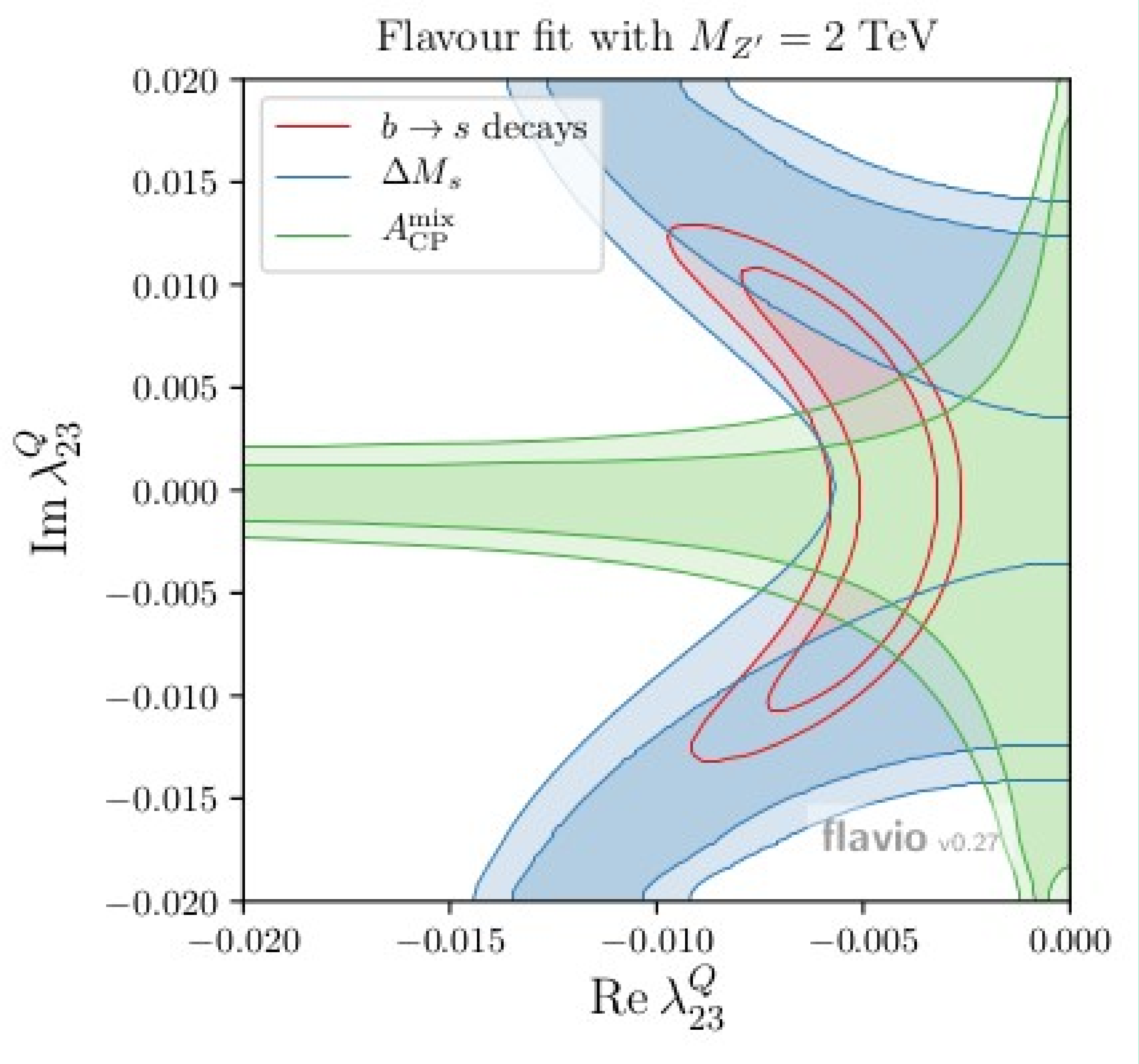
  from lattice / sum rules – reduce error in $\Delta\Gamma_s$

- Lattice confirmation of dimension-6

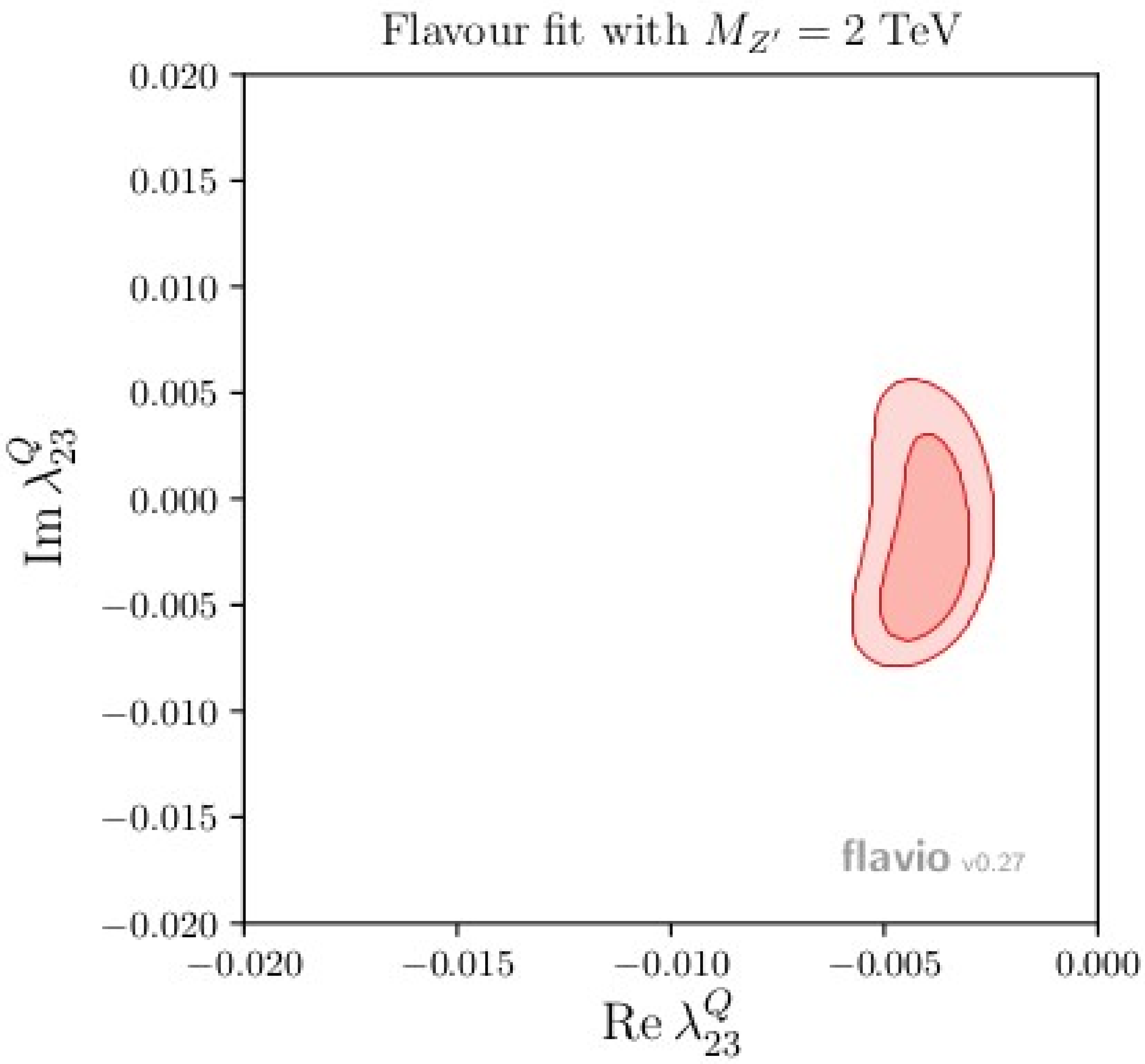
- Know $\tau(B_s)/\tau(B_d)$ better from experiment – while already doing very well, theory is currently ahead

Thanks!

Backup



Flavour fit with $M_{Z'} = 2$ TeV
$b \to s$ decays
ΔM_s
$A_{\rm CP}^{\rm mix}$
0.020
0.015
0.010
0.005
0.000
−0.005
−0.010
−0.015
−0.020
$\mathrm{Im}\,\lambda_{23}^{Q}$
−0.020
−0.015
−0.010
−0.005
0.000
$\mathrm{Re}\,\lambda_{23}^{Q}$
flavio v0.27



Flavour fit with $M_{Z'} = 2$ TeV
0.020
0.015
0.010
0.005
0.000
−0.005
−0.010
−0.015
−0.020
$\mathrm{Im}\,\lambda^Q_{23}$
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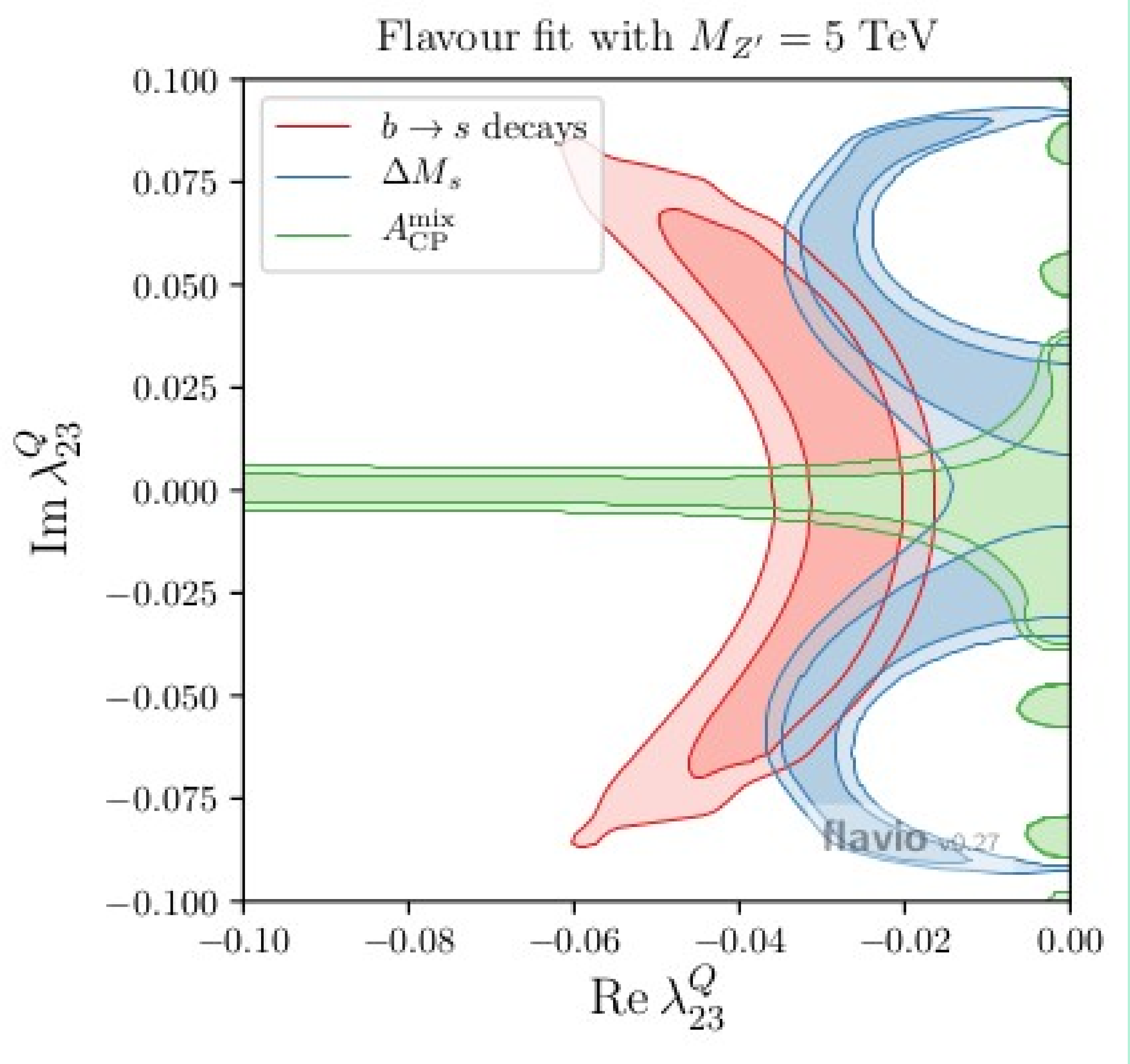
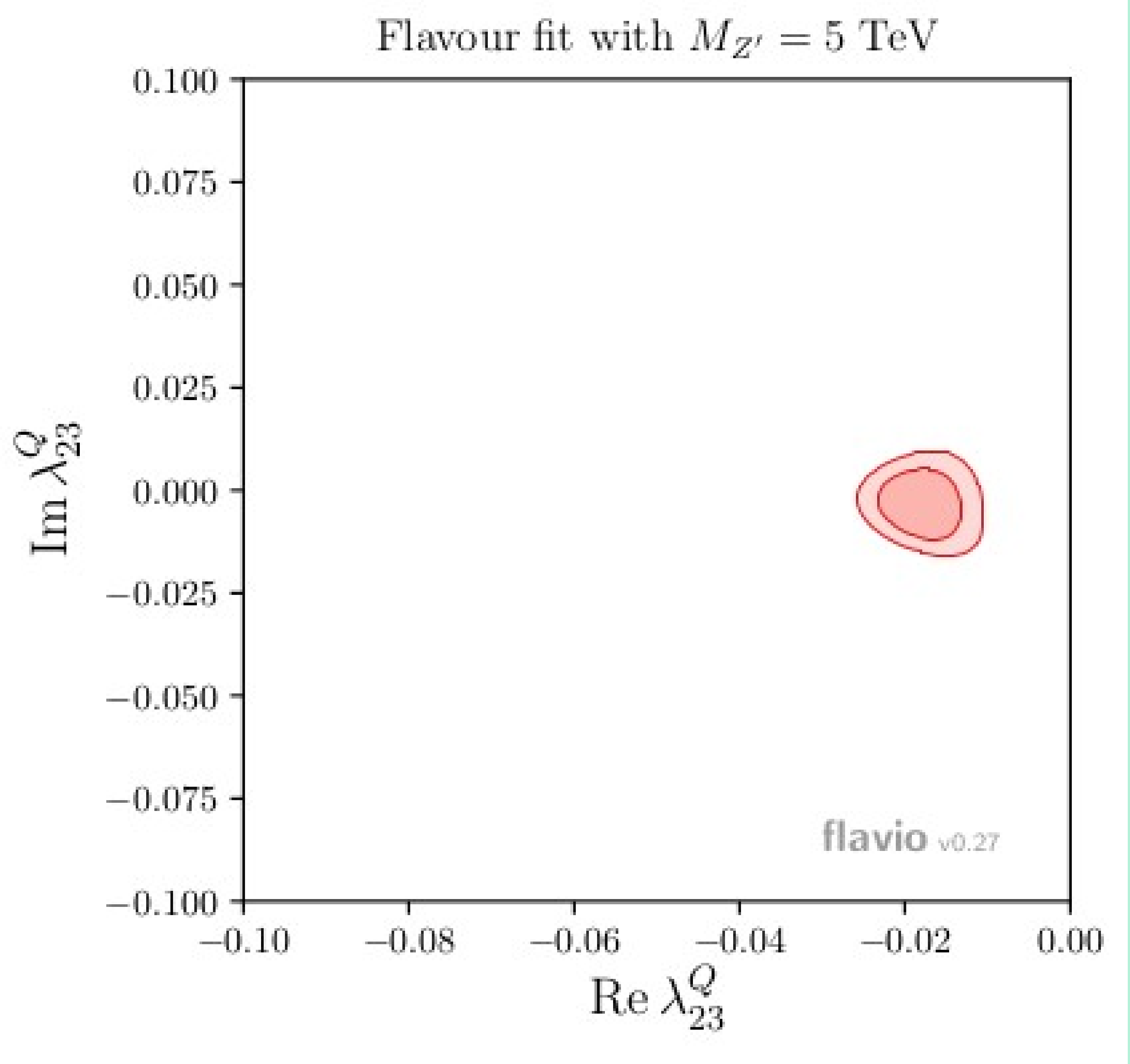
flavio v0.27


Flavour fit with $M_{Z'} = 5$ TeV
$b \to s$ decays
ΔM_s
$A_{\mathrm{CP}}^{\mathrm{mix}}$
0.100
0.075
0.050
0.025
0.000
−0.025
−0.050
−0.075
−0.100
−0.10
−0.08
−0.06
−0.04
−0.02
0.00
$\mathrm{Im}\,\lambda_{23}^{Q}$
$\mathrm{Re}\,\lambda_{23}^{Q}$
flavio v0.27



Flavour fit with $M_{Z'} = 5$ TeV
0.100
0.075
0.050
0.025
0.000
−0.025
−0.050
−0.075
−0.100
$\mathrm{Im}\,\lambda_{23}^{Q}$
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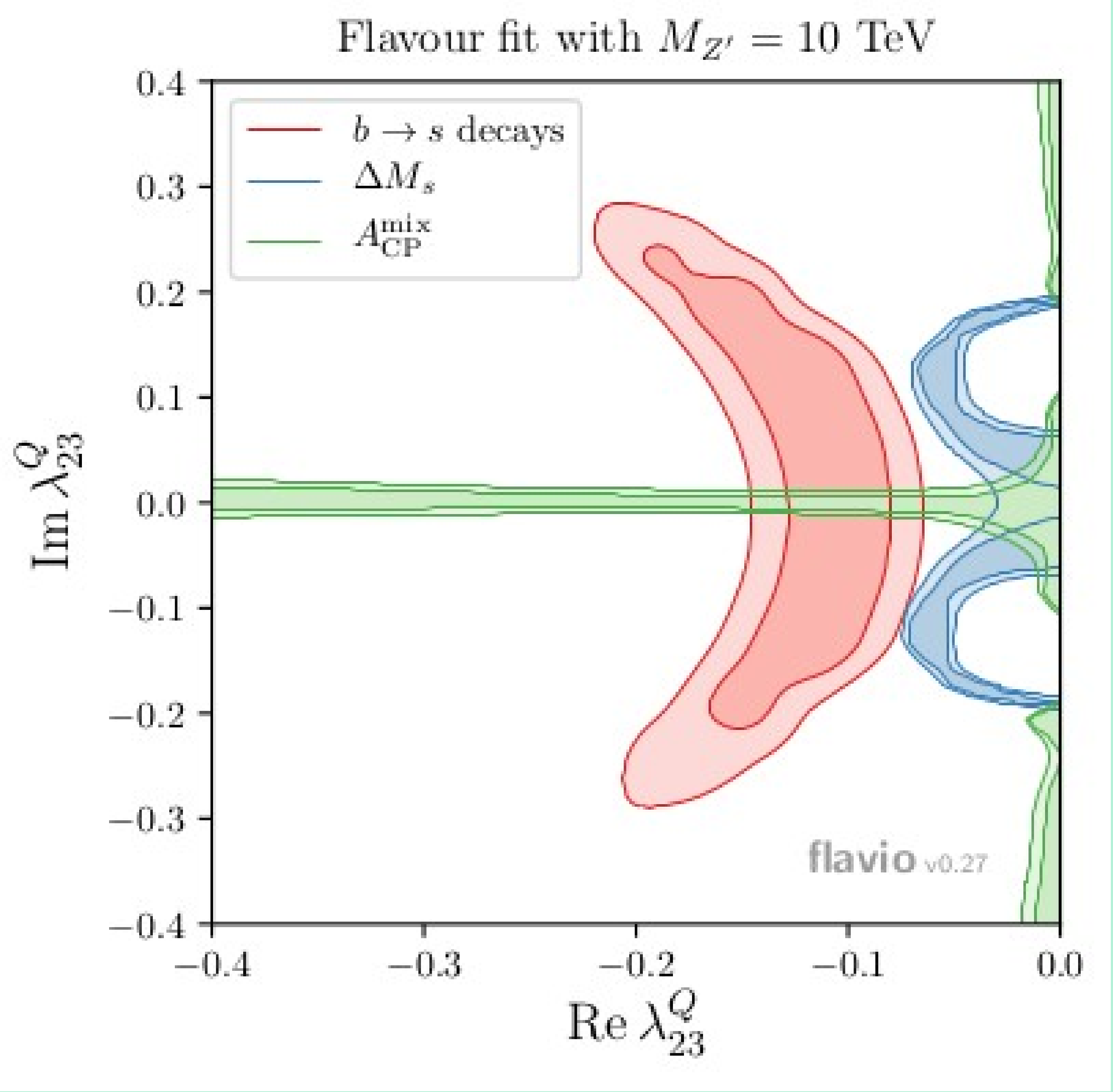
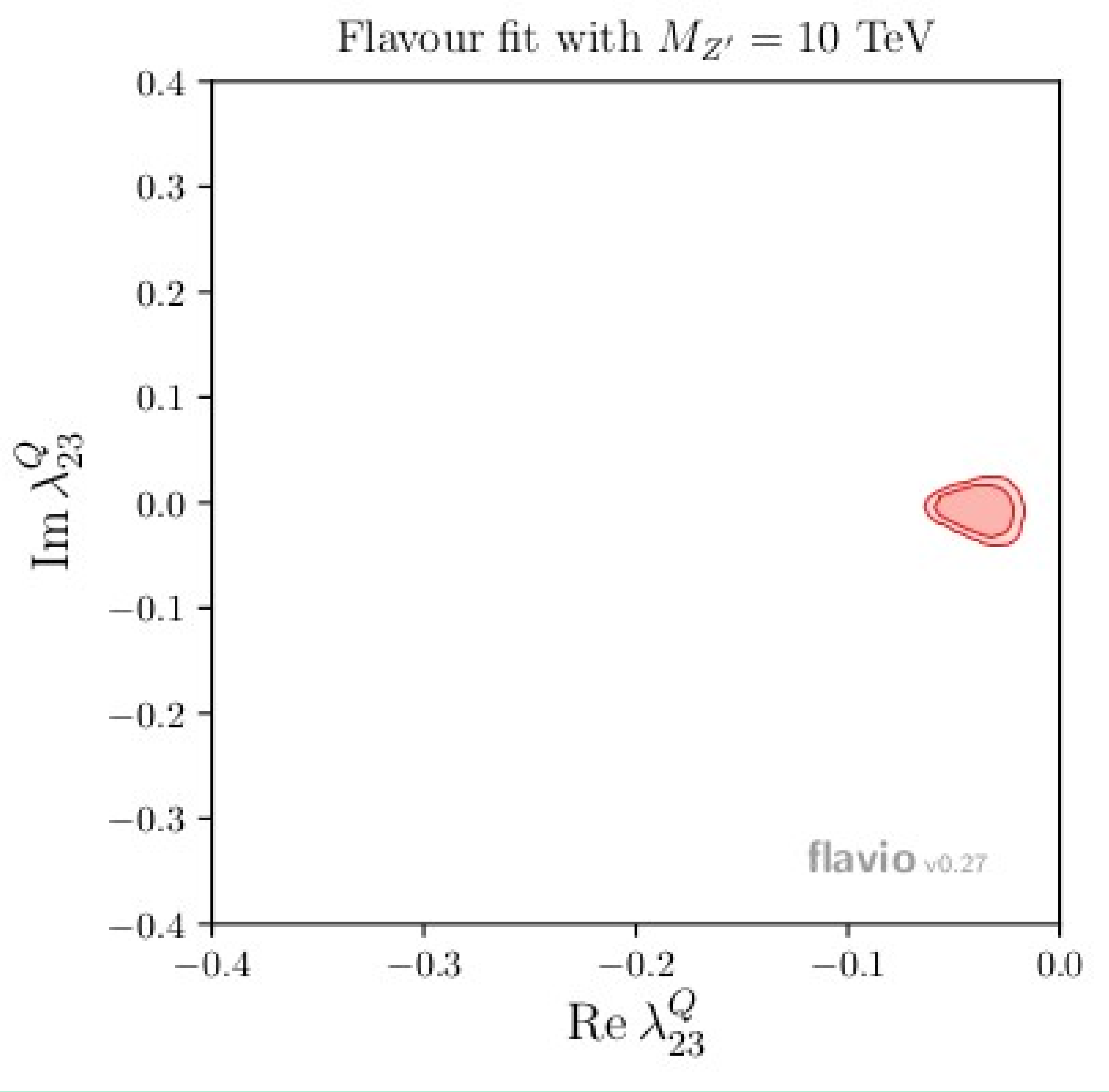


Flavour fit with $M_{Z'} = 10$ TeV
$b \to s$ decays
ΔM_s
$A_{\mathrm{CP}}^{\mathrm{mix}}$
0.4
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
−0.4
−0.3
−0.2
−0.1
0.0
$\mathrm{Im}\,\lambda_{23}^{Q}$
$\mathrm{Re}\,\lambda_{23}^{Q}$
flavio v0.27



Flavour fit with $M_{Z'} = 10$ TeV
0.4
0.3
0.2
0.1
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−0.3
−0.4
$\mathrm{Im}\,\lambda_{23}^{Q}$
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−0.3
−0.2
−0.1
0.0
$\mathrm{Re}\,\lambda_{23}^{Q}$
flavio v0.27

Mixing operators

$$Q_1 = \bar{b}_i \gamma_\mu (1-\gamma^5) q_i \; \bar{b}_j \gamma^\mu (1-\gamma^5) q_j,$$

$$Q_2 = \bar{b}_i (1-\gamma^5) q_i \; \bar{b}_j (1-\gamma^5) q_j, \qquad Q_3 = \bar{b}_i (1-\gamma^5) q_j \; \bar{b}_j (1-\gamma^5) q_i,$$

$$Q_4 = \bar{b}_i (1-\gamma^5) q_i \; \bar{b}_j (1+\gamma^5) q_j, \qquad Q_5 = \bar{b}_i (1-\gamma^5) q_j \; \bar{b}_j (1+\gamma^5) q_i,$$

Lifetime operators

$$Q_1^q = \bar{b}\gamma_\mu(1-\gamma^5)q\ \bar{q}\gamma^\mu(1-\gamma^5)b, \qquad T_1^q = \bar{b}\gamma_\mu(1-\gamma^5)T^A q\ \bar{q}\gamma^\mu(1-\gamma^5)T^A b,$$

$$Q_2^q = \bar{b}(1-\gamma^5)q\ \bar{q}(1+\gamma^5)b, \qquad T_2^q = \bar{b}(1-\gamma^5)T^A q\ \bar{q}(1+\gamma^5)T^A b.$$

HQET sum rules – results

D mixing