Hadronic tau decays and the determination of V_{us}

Beyond the Flavour Anomalies 2026

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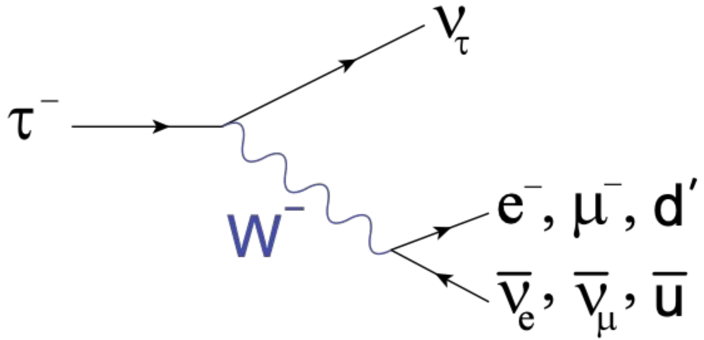
Tau decays

Tau lepton has a mass around 1776.9 MeV/c^2

The only lepton heavy enough to have hadronic decay modes

$|d'\rangle = V_{ud}\,|d\rangle + V_{us}\,|s\rangle$



Final state containing net strangeness ($|S|=1$) relevant for $|V_{us}|$ determination

Two types of measurements:

- Branching fraction (normalisation)
- Spectral function or mass spectrum (shape)

Branching fraction measurements

ALEPH provides the most comprehensive measurement in 1999

Decay	K^0 detected	S	B (10^{-3})
$\tau^- \to K^-\nu_\tau$	–		6.96 ± 0.29
$\tau^- \to K^-\pi^0\nu_\tau$	–		4.44 ± 0.35
$\tau^- \to \overline{K^0}\pi^-\nu_\tau$	K_L^0		9.28 ± 0.56
$\tau^- \to \overline{K^0}\pi^-\nu_\tau$	K_S^0		8.55 ± 1.34
$\tau^- \to \overline{K^0}\pi^-\pi^0\nu_\tau$	K_L^0		3.47 ± 0.65
$\tau^- \to \overline{K^0}\pi^-\pi^0\nu_\tau$	K_S^0		2.94 ± 0.82
$\tau^- \to K^-\pi^+\pi^-\nu_\tau$	–	-1	2.14 ± 0.47
$\tau^- \to K^-\pi^0\pi^0\nu_\tau$	–		0.56 ± 0.25
$\tau^- \to \overline{K^0}\pi^-\pi^0\pi^0\nu_\tau$	K_L^0		< 0.66 (95% C.L.)
$\tau^- \to \overline{K^0}\pi^-\pi^0\pi^0\nu_\tau$	K_S^0		0.58 ± 0.36
$\tau^- \to K^-\pi^0\pi^0\pi^0\nu_\tau$	–		0.37 ± 0.24 (excl. η)
$\tau^- \to K^-\pi^+\pi^-\pi^0\nu_\tau$	–		0.54 ± 0.43 (excl. η)
$\tau^- \to K^-\eta\nu_\tau$	–		$0.29^{+0.15}_{-0.14}$
$\tau^- \to K^-K^+K^-\nu_\tau$	–		< 0.19 (95% C.L.)

Measured channels containing *K* but with S=0 not shown here

The dominant channel is measured with a relative precision of 4.2%

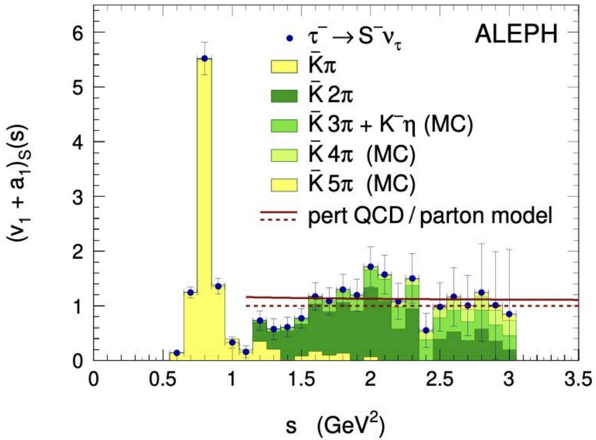
Branching fraction measurements

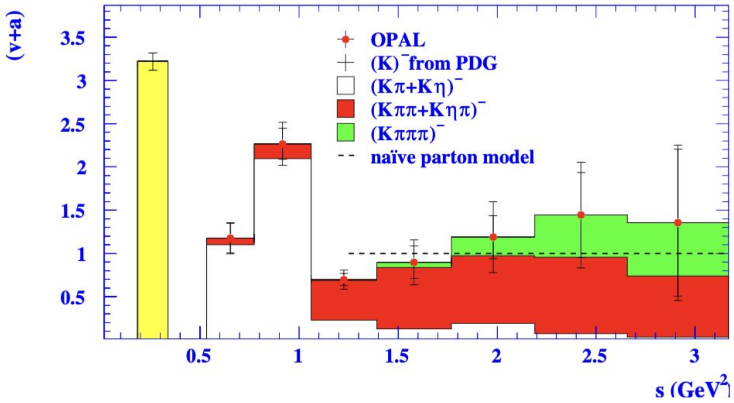
The most precisely measured channels used in the $|V_{us}|$ extraction

Channel	Best measurement ($\times 10^{-3}$)	HFLAV fit ($\times 10^{-3}$)	PDG average ($\times 10^{-3}$)
$\tau^- \to K^- \nu_\tau$	6.96 (4.2%) [ALEPH'99, $\sim 200\,\mathrm{pb}^{-1}$]	6.97 (1.4%)	6.85 (3.4%)
$\tau^- \to K_S^0 \pi^- \nu_\tau$	4.16 (1.9%) [Belle'14, $669\,\mathrm{fb}^{-1}$]	4.19 (1.7%)	4.195 (2.6%)
$\tau^- \to K^- \pi^0 \nu_\tau$	4.16 (4.4%) [BABAR'07, $230\,\mathrm{fb}^{-1}$]	4.33 (3.5%)	4.26 (3.8%)
$\tau^- \to X_s^- \nu_\tau$		29.2 (1.4%)	

Strange spectral function measurements

Early measurements by ALEPH and OPAL





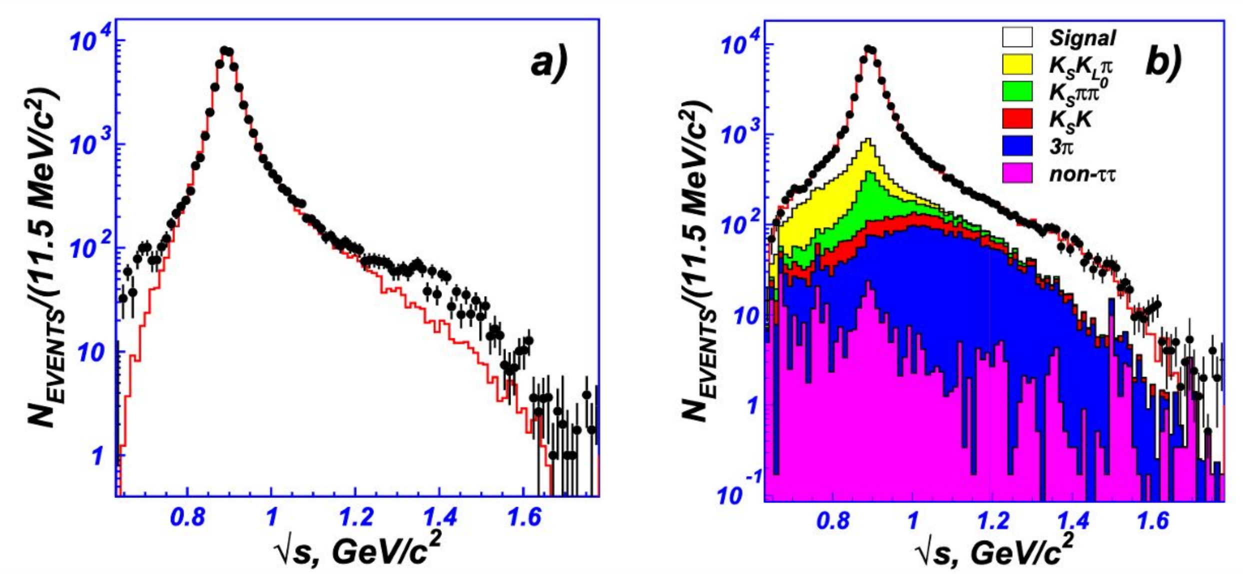
OPAL normalised each channel with branching fraction from PDG, dominated by ALEPH

In ALEPH's version, the dominant axial-vector channel $\tau^- \to K^- \nu_\tau$ not shown

Precision very limited, some unmeasured channels included according to MC expectation

Strange spectral function measurements

Improved measurement in the $\tau^- \to K^0{}_S \pi^- \nu_\tau$ channel by Belle (2007, 351 fb^{-1})



Extraction of $|V_{us}|$ from $\mathcal{B}(\tau^- \to K^- \nu_\tau)$

$$\mathcal{B}(\tau^- \to K^- \nu_\tau) = \frac{G_F^2}{16\pi} f_K^2 |V_{us}|^2 \tau_\tau m_\tau^3 \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 S_{\mathrm{EW}}(1+\delta)$$

Most uncertain inputs are

$\mathcal{B}(\tau^- \to K^- \nu_\tau) = 6.97 \cdot 10^{-3} (1.4\%)$ [HFLAV fit], it contributes 0.70%

$\delta = (-0.15 \pm 0.57)\%$ [arXiv:2107.04603], it contributes 0.28%

$f_K = 155.7 \pm 0.3\,\mathrm{MeV}$ [Lattice QCD, HFLAV], it contributes 0.19%

➔ $|V_{us}| = 0.2224$ (0.76%)

Extraction of $|V_{us}|$ from $\mathcal{B}(\tau^- \to K^- \nu_\tau)/\mathcal{B}(\tau^- \to \pi^- \nu_\tau)$

$$\frac{\mathcal{B}(\tau^- \to K^- \nu_\tau)}{\mathcal{B}(\tau^- \to \pi^- \nu_\tau)} = \frac{f_K^2 |V_{us}|^2}{f_\pi^2 |V_{ud}|^2} \frac{(m_\tau^2 - m_K^2)^2}{(m_\tau^2 - m_\pi^2)^2} (1 + \delta')$$

Most uncertain inputs are

$\mathcal{B}(\tau^- \to K^- \nu_\tau)/\mathcal{B}(\tau^- \to \pi^- \nu_\tau) = 6.44 \cdot 10^{-3} (1.4\%)$ [HFLAV fit], it contributes 0.70%

$\delta' = (0.10 \pm 0.80)\%$ [arXiv:2107.04603], it contributes 0.40%

$f_K/f_\pi = 1.1934 \pm 0.0019\,\mathrm{MeV}$ [Lattice QCD, HFLAV], it contributes 0.16%

➔ $|V_{us}| = 0.2229$ (0.85%)

Extraction of $|V_{us}|$ from inclusive strange spectral functions

There are several ways to extract $|V_{us}|$ using the inclusive strange spectral functions depending on the theoretical predictions to be used:

1. Flavour-breaking finite-energy sum rule (operator product expansion)
2. Lattice QCD calculations

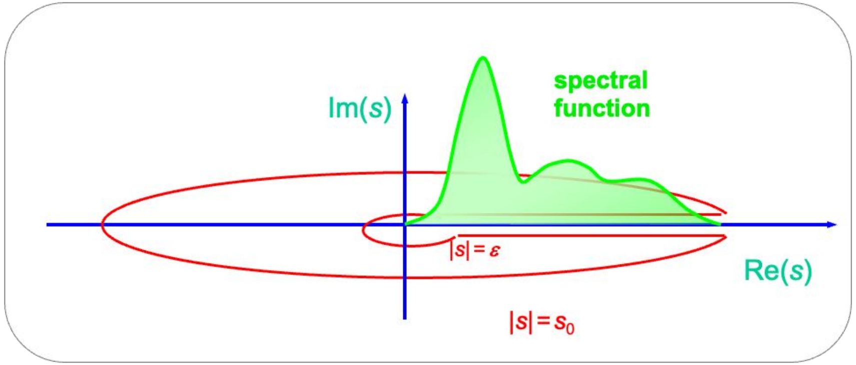
One example is shown here in the following two slides for case 1

Extraction of $|V_{us}|$ from inclusive strange spectral functions

hep-hp/0507078

$$R(s_0) \propto \int_0^{s_0} ds w(s) \mathrm{Im}\Pi(s+i\epsilon) \Leftrightarrow -\frac{1}{2i}\oint_{s=s_0} ds w(s)\Pi(s)$$

$$R_{\tau,S} = 3|V_{us}|^2 S_{\mathrm{EW}} \left(1 + \delta^{(0)} + \delta'_{\mathrm{EW}} + \delta_S^{(2,m_q)} + \sum_{D=4,6,\ldots} \delta_S^{(D)}\right)$$



$\delta^{(0)}_{\mathrm{pQCD}}(\alpha_s)$ Dominant pQCD contribution, expansion known up to 5th order

$\delta'_{\mathrm{EW}} \simeq 0.0010$ Non-logarithmic EW correction (Braaten-Li, 1990)

$\delta^{(2)}(m_q)$ Dimension 2 pQCD contribution from quark mass (<0.1% for *u*, *d* quarks)

$\delta^{(D)}_{\text{non-pQCD}}$ Dimension D non-pQCD contributions

$$\delta_S^{(2,m_s)} = -8\frac{m_s^2(m_\tau^2)}{m_\tau^2}\left[1 + \frac{16}{3}a_s + 46.00\,a_s^2 + \left(283.6 + \frac{3}{4}x_3^{(1+0)}\right)a_s^3 + \cdots\right]$$

Extraction of $|V_{us}|$ from inclusive strange spectral functions

In addition to the inclusive observable $R(s)$, define several other observables by different moments of the spectral functions

$$R^{k\ell}_{\tau,(S)} = \int_0^{m_\tau^2} ds \left(1 - \frac{s}{m_\tau^2}\right)^k \left(\frac{s}{m_\tau^2}\right)^\ell \frac{dR_{\tau,(S)}}{ds}$$

One can then fit several unknown parameters including $|V_{us}|$ and m_s

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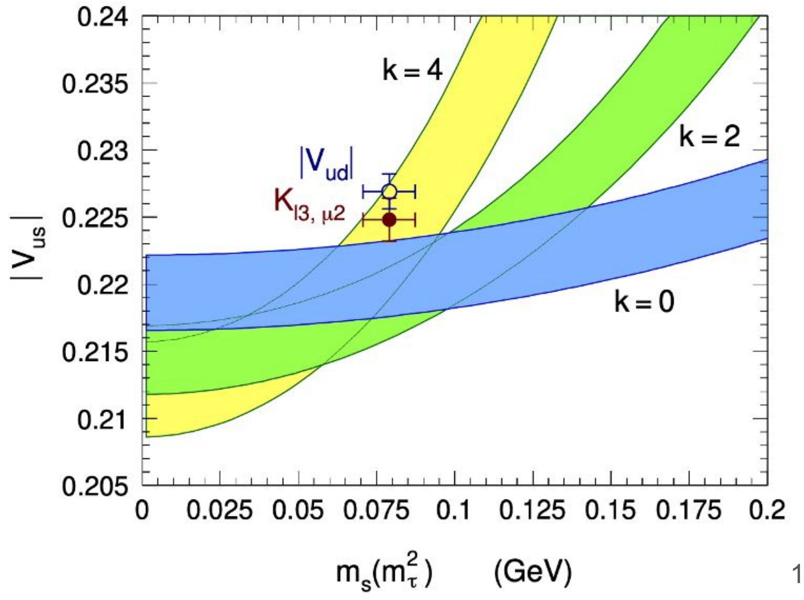


Prospects

On the experimental side: improve precision on the branching fractions and spectral functions:

Using >365 fb^{-1} current Belle II data sample, the following analyses are on going:

- $\tau^- \to K^- \pi^0 \nu_\tau$
- $\tau^- \to K^- \pi^+ \pi^- \nu_\tau$
- $\tau^- \to K^- \eta \nu_\tau$
- $\tau^- \to K^- \eta \pi^0 \nu_\tau$ (487 fb^{-1})
- $\tau^- \to K^- \pi^+ \pi^- \pi^0 \nu_\tau$
- $\tau^- \to K^0_S \pi^- \nu_\tau$
- Inclusive strange decays $\tau^- \to K^- X_{no\text{-}s} \nu_\tau$, $K^0_S X_{no\text{-}s} \nu_\tau$, $K^0_L \pi^- X_{no\text{-}s} \nu_\tau$

The high multiplicity decay modes should improve the precision for the spectral function tail towards the tau mass limit

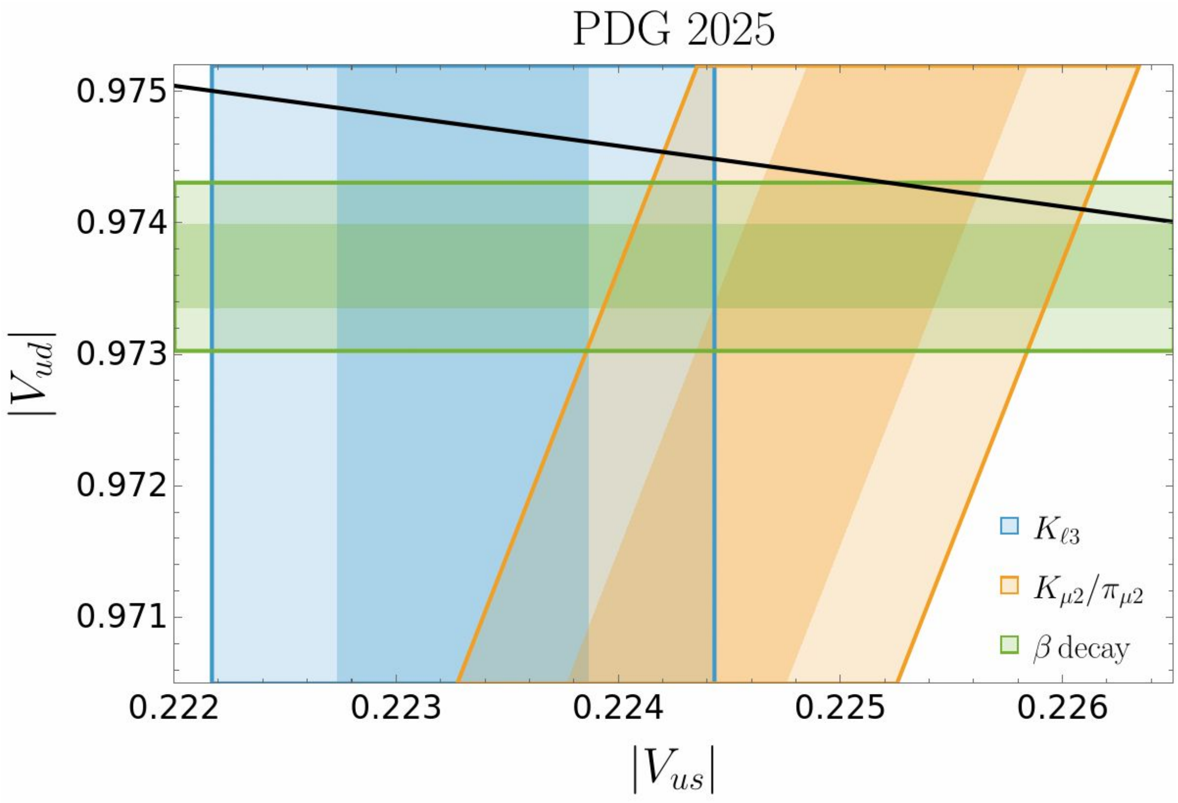
On the theoretical side: improve radiative correction calculations

Why are "we" interested in V_{us} determinations?

- In SM, CKM is a 3x3 unitary matrix
- Implies many relationships between elements
- Including "first row unitarity":
  - $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

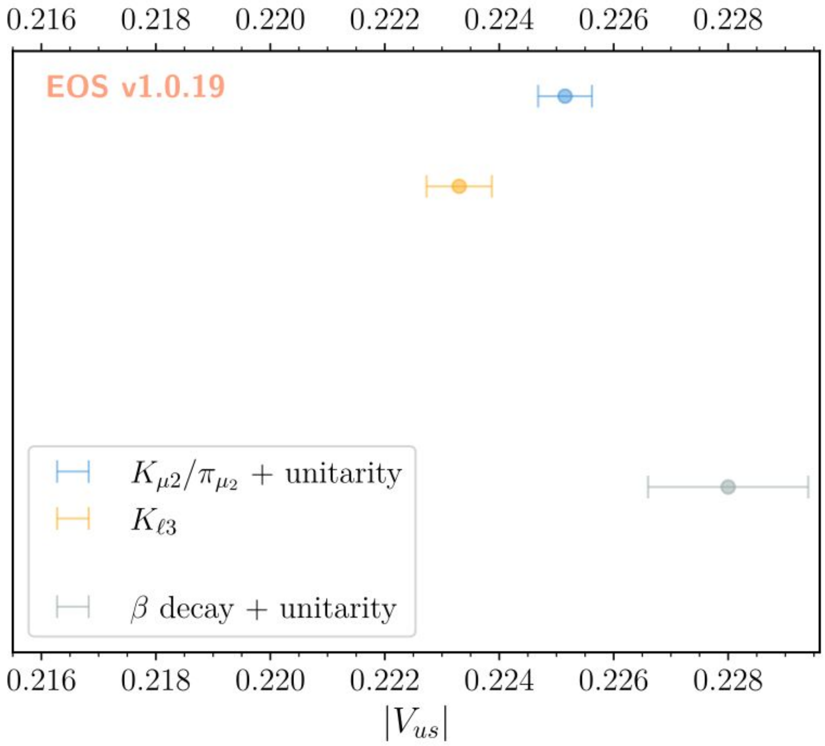
$|V_{ub}|^2 \sim 10^{-5}$

Cabibbo angle anomaly overview



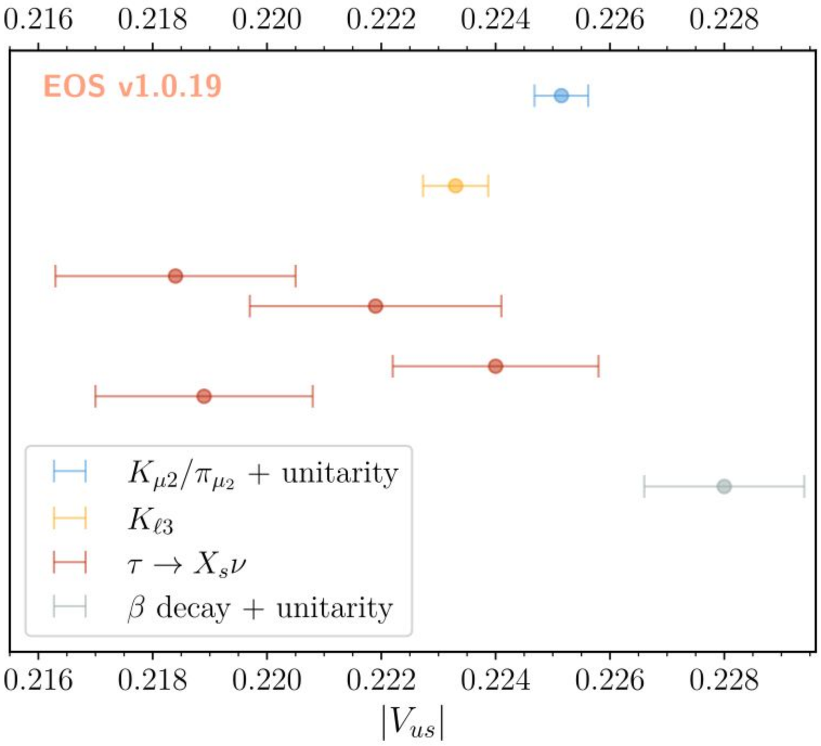
Cabibbo angle anomaly overview

- In 2 flavour paradigm, this is equivalent to the statement that there is a single Cabibbo angle that controls 1st & 2nd gen flavour rotations
- $V_{us} \approx \sin\theta_C$, $V_{ud} \approx \cos\theta_C$, $V_{us}/V_{ud} \approx \tan\theta_C$



Cabibbo angle anomaly overview

- Clear discrepancy also when including taus
  - Vital to combine all available data
- Three different strange decays ($K_{\ell 2}$, $K_{\ell 3}$, $\tau \to X_s \nu$) give different answers



Hadronic description of s->u transitions

- Consider $j^V = \bar{u}\gamma^\mu s\,, j^A = \bar{u}\gamma^\mu\gamma^5 s$
- $K_{\ell 2}$ depends on $\langle 0|j^A|K\rangle \sim f_K$
  - Simple, well known from lattice, but enters theory prediction for f_0
- $K_{\ell 3}$ depends on $\langle \pi|j^V|K\rangle \sim f_{+,0}(q^2)$ with $0 \leq q^2 \leq (m_K - m_\pi)^2$
  - Lattice can do $f_+(0) = f_0(0)$, but q^2 dependence traditionally fitted to experimental data
- $\tau \to X_s\nu$ also depends on $f_{+,0}(q^2)$ but now with $(m_K + m_\pi)^2 \leq q^2 \leq m_\tau^2$

q^2 dependence of form factor

- Taylor expand: $f(q^2) = f(0) + \lambda \frac{q^2}{m_\pi^2} + \cdots$
  - Not valid for large q^2, cannot be used for tau decay
- Dispersion model: $f(q^2) = f(0) \times \exp\left(q^2 \times (C + G(q^2))\right)$
  - C is constant, G is dispersive integral over phase of f up to infinity
  - Valid for tau, but dispersive integral needs assumptions about phase

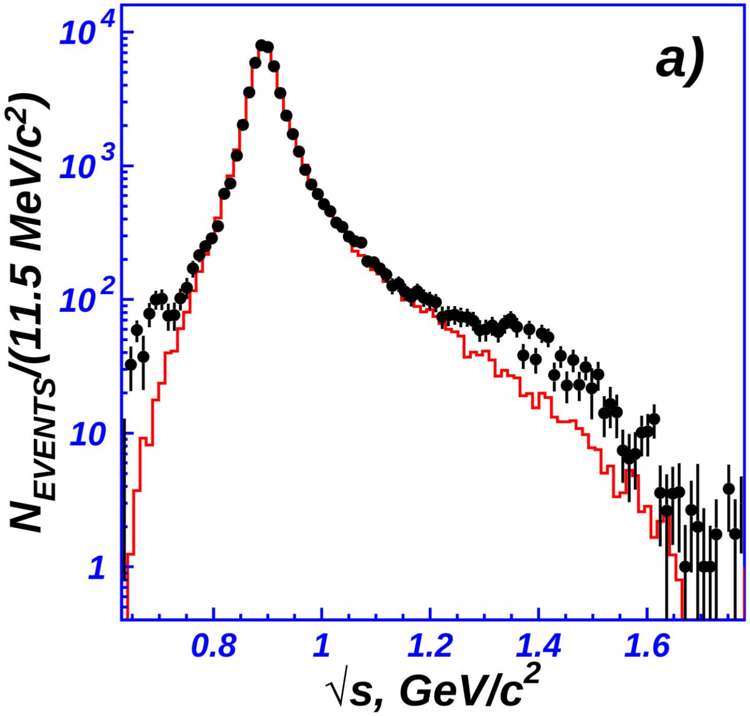
New form factor parameterisation

- For semi-leptonic FFs, BGL style FF is $f(q^2) = (1/\phi) \sum_i a_i z(q^2)^i$
  - z-mapping to construct dispersively bounds
- But for q^2 above threshold, pole structure has to be fitted through z polynomial, not easy / sensible
- We extend to $f(q^2) = (W/\phi) \prod_r \frac{1}{z - z_r} \frac{1}{z - z_r^*} \sum_i b_i z^i$
  - W regularises behaviour at threshold/infinity
  - Explicit resonance poles (in z)

Fitting FF shape / poles

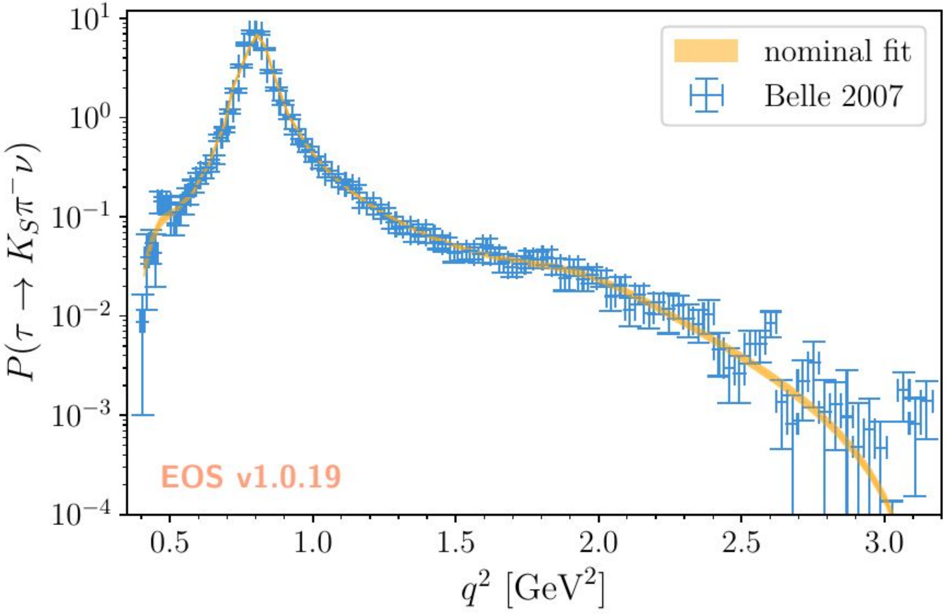
- We fit q^2 differential and total BR data for tau and K decays
  - Only depends on $|FF|^2$
  - No phase information enters
- Differential tau data only available from Belle
  - And only 2007 data, 2014 given in figure but not numerically

Belle 2007



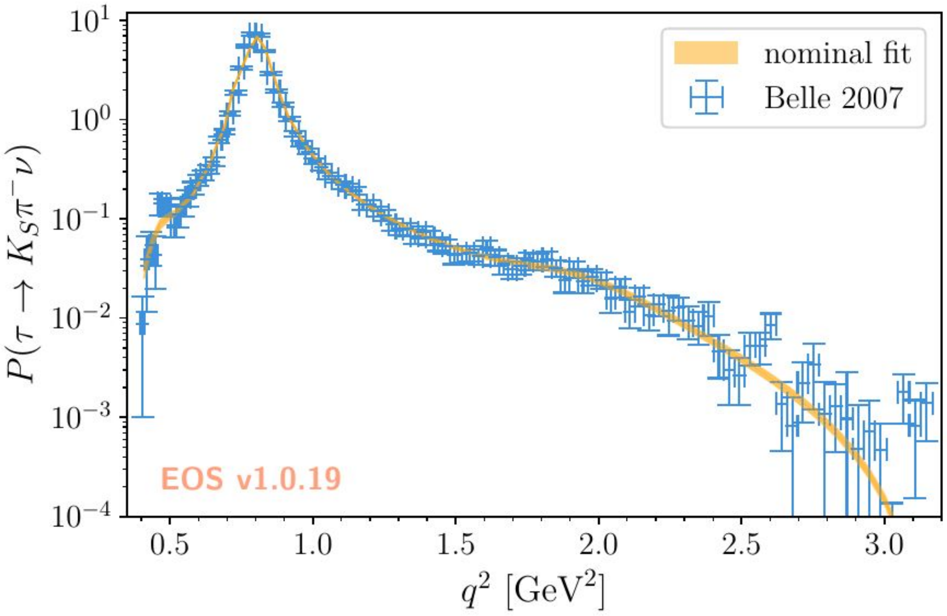
Fitting FF shape & poles

- Tried fitting purely from data and theory (not lattice)
- $K^*(892)$ easy



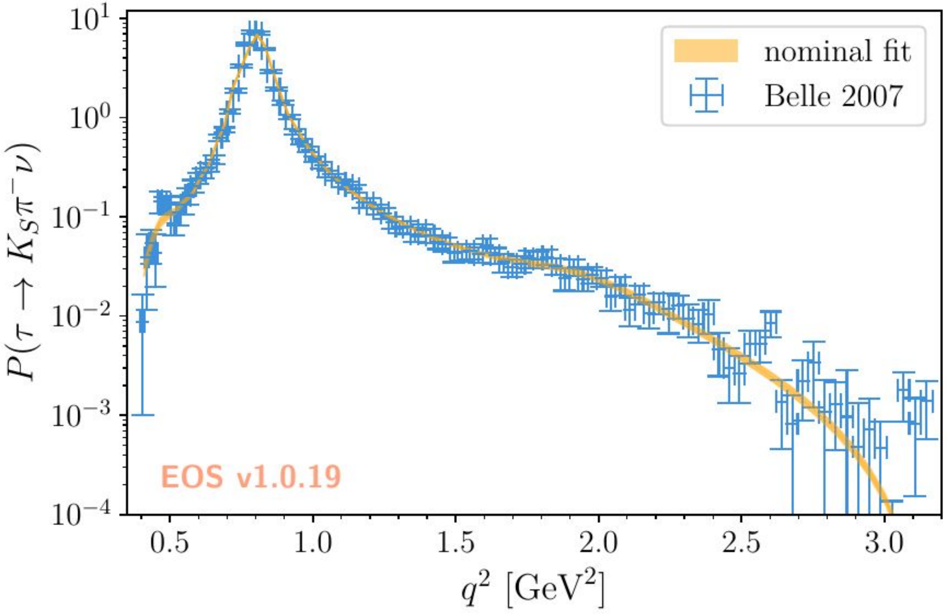
Fitting FF shape & poles

- $K^{0*}(700)$ and $K^{*}(1680)$ hard - probably because close to endpoints
- Question: Data at $q^2 \sim m_\tau^2$ from experiment?



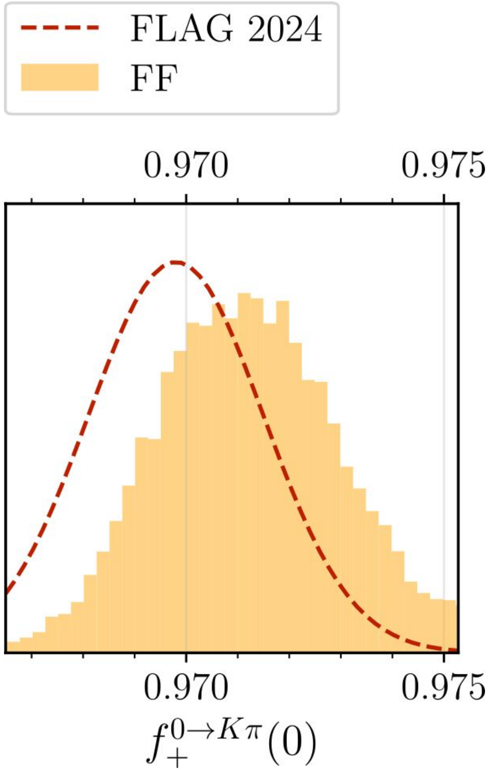
Fitting FF shape & poles

- Could fit $K^*(1410)$ *or* $K^{0*}(1430)$ but not both - probably due to not having an angular information
- Question: Angular info from experiment?



Comparing fit with known results

- Using PDG for all except $K^*(892)$, we find good agreement with :
    - PDG $K^*(892)$ mass/width
    - Lattice $f_+(0)$
    - Taylor expansion fit to slope at $q^2=0$



Bonus fit result - inputs for hadronic D decays

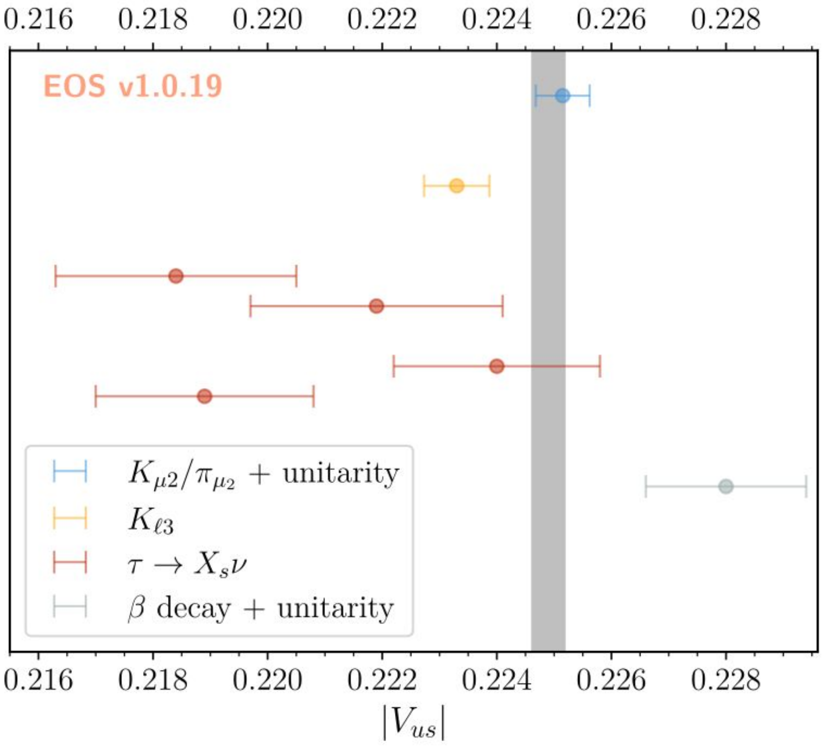
- Hadronic D decays, topological amplitudes are leading order in $1/N_c$ expansion depend on $f_0(q^2 = m_D^2)$
- Previous work used $1 < |f_0(q^2 = m_D^2)| < 4.5$
- Our fit gives $1 < |f_0(q^2 = m_D^2)| < 2.4$

Global determination of V_{us}

- Now include lattice $f_+(0)$

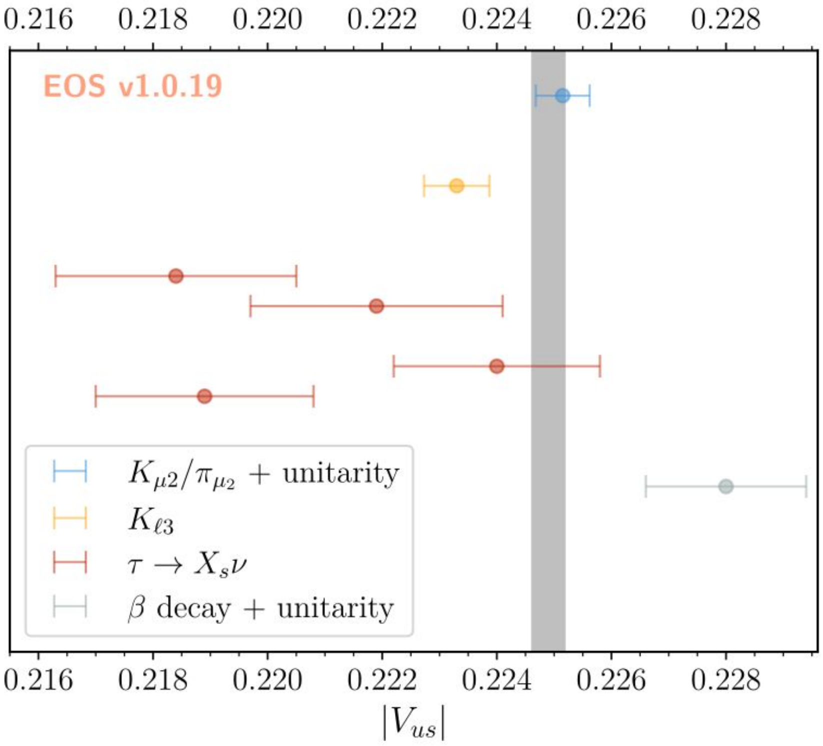
Global determination of V_{us}

- Now include lattice $f_+(0)$
- Get V_{us} = 0.2244 +- 0.0005 as nominal result
  - Note we do not include isospin breaking or EM effects, result is mostly proof of concept



Global determination of V_{us}

- Now include lattice $f_+(0)$
- Get V_{us} = 0.2244 +- 0.0005 as nominal result
- Implies continued unitarity violation of 2σ



BSM as an explanation?

- Previous work suggests BSM in the form of RH currents, i.e. a current of the form $\bar{u}\gamma^{\mu}P_R s$ is favoured
- We studied this:
  - WC best fit value agrees with literature
  - But also consistent with 0 at 1σ
    - And of course we neglect isospin/EM effects

Conclusions & questions

- Tau decays offer additional window on V_{us} and Cabibbo Angle Anomaly
- New parameterisation developed that manifestly links kaon and tau decays, while relying on minimal assumptions
- Looking forward to taking into account EM and isospin breaking corrections everywhere

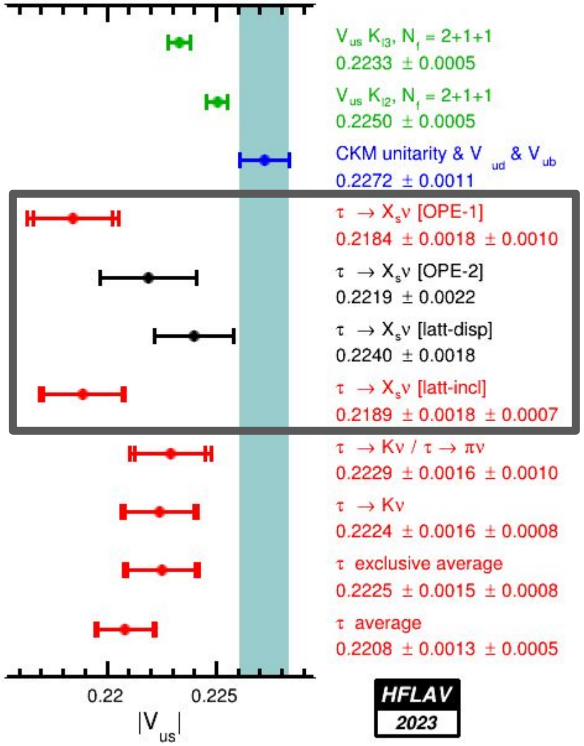
Conclusions & questions

1) Can (/ when will) experiments provide differential and angular BR data?
2) Are there any unexpected / unknown corrections?
3) Can / will we get to 5σ for BSM?

BACKUP

HFLAV tau Vus analysis / SciPost Phys. Proc. 1, 006 (2019)

- "OPE-1" / "OPE-2"
  - Choices for w, s0 in $\int_0^{s_0} w(s)\rho(s)\,ds = -\frac{1}{2\pi i}\oint_{|s|=s_0} w(s)\Pi(s)\,ds$
- "latt-disp"
  - Lattice calculates "us" HVP at unphysical q^2, relate to experimental $d\Gamma(\tau \to X_s\nu)/dq^2$ through dispersion relation
- "latt-incl"
  - Lattice calculate $\mathcal{B}(\tau \to X_s\nu)$ / $|V_{us}|^2$, combine with exp measurement of $\mathcal{B}(\tau \to X_s\nu)$



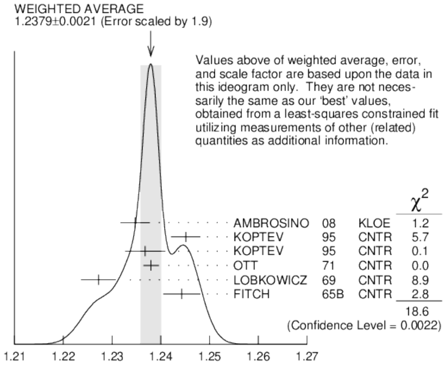
Tensions in kaon measurements

Scrutinizing CKM unitarity with a new measurement of the $K_{\mu 3}/K_{\mu 2}$ branching fraction

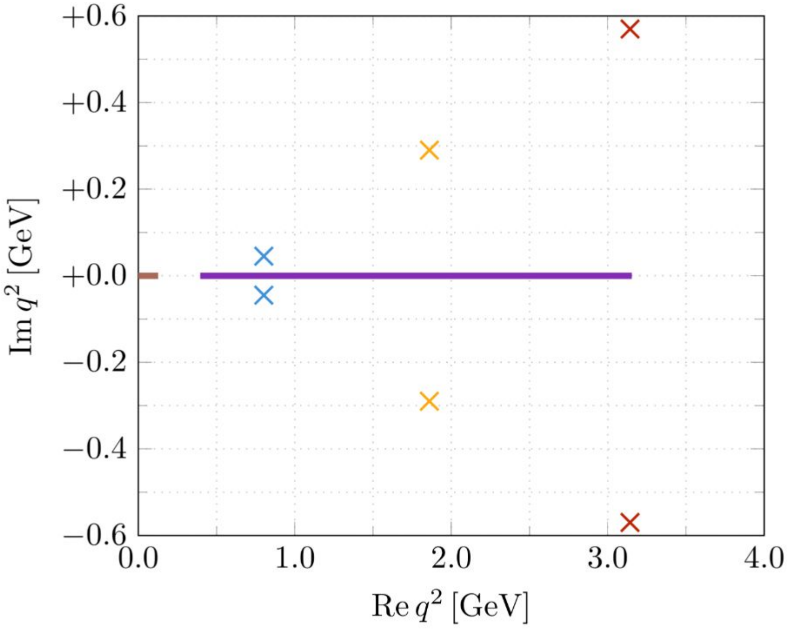
Vincenzo Cirigliano[a], Andreas Crivellin[b,c], Martin Hoferichter[d], Matthew Moulson[e]

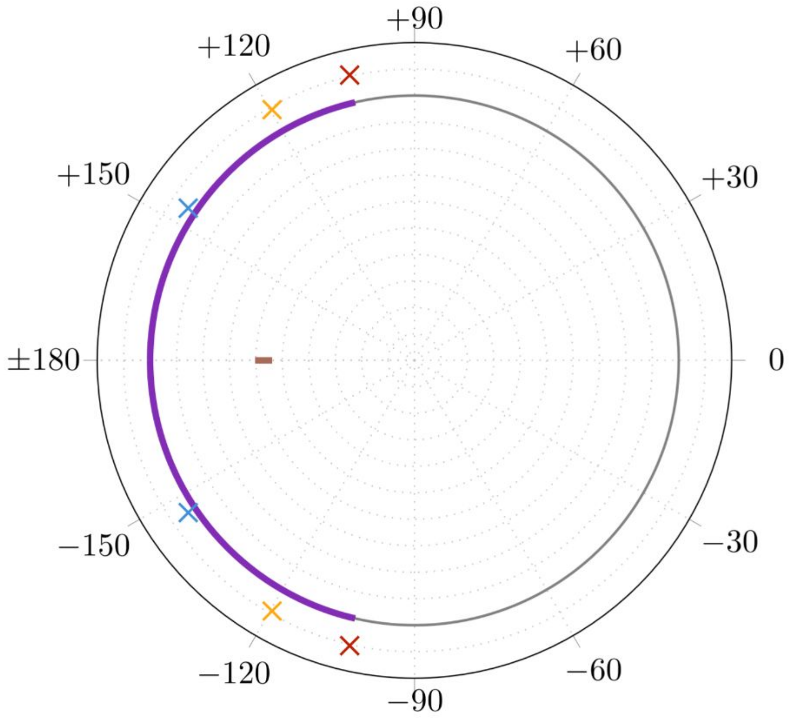
the experimental situation in the kaon sector, especially in view of tensions in the global fit to kaon data as well as the fact that the $K_{\mu 2}$ channel is currently dominated by a single experiment. Such a measurement, as possible for example at NA62, would further provide important constraints on physics beyond the Standard Model, most notably on the role of right-handed vector currents.

NA62 has done this - now awaiting analysis!



z-parameterisation





Taylor expansion

$$f_{+,0}(t) = f_{+,0}(0)\left(1 + \frac{\lambda'_{+,0}t}{m_\pi^2} + \frac{\lambda''_{+,0}t^2}{2m_\pi^4}\right)$$

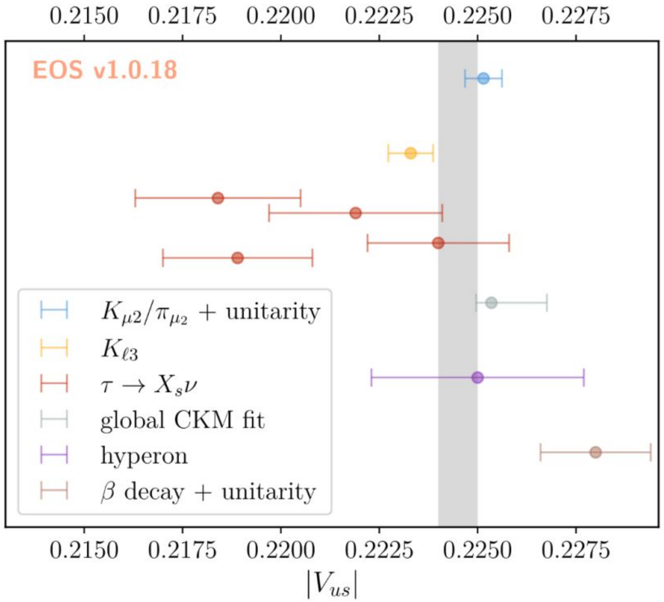
$$\lambda'_+\Big|_{\mathrm{NA48/2}}^{\text{quadratic model}} = 0.024\,24 \pm 0.000\,75\ (\text{stat}) \pm 0.001\,30\ (\text{sys})\,,$$
$$\lambda'_0\Big|_{\mathrm{NA48/2}}^{\text{linear model}} = 0.014\,47 \pm 0.000\,63\ (\text{stat}) \pm 0.001\,17\ (\text{sys})\,,$$

with our posterior prediction

$$\lambda'_+ = 0.0238 \pm 0.0007\ (\text{stat}) \pm 0.0020\ (\text{model})\,,$$
$$\lambda'_0 = 0.0119 \pm 0.0011\ (\text{stat}) \pm 0.0010\ (\text{model})\,.$$

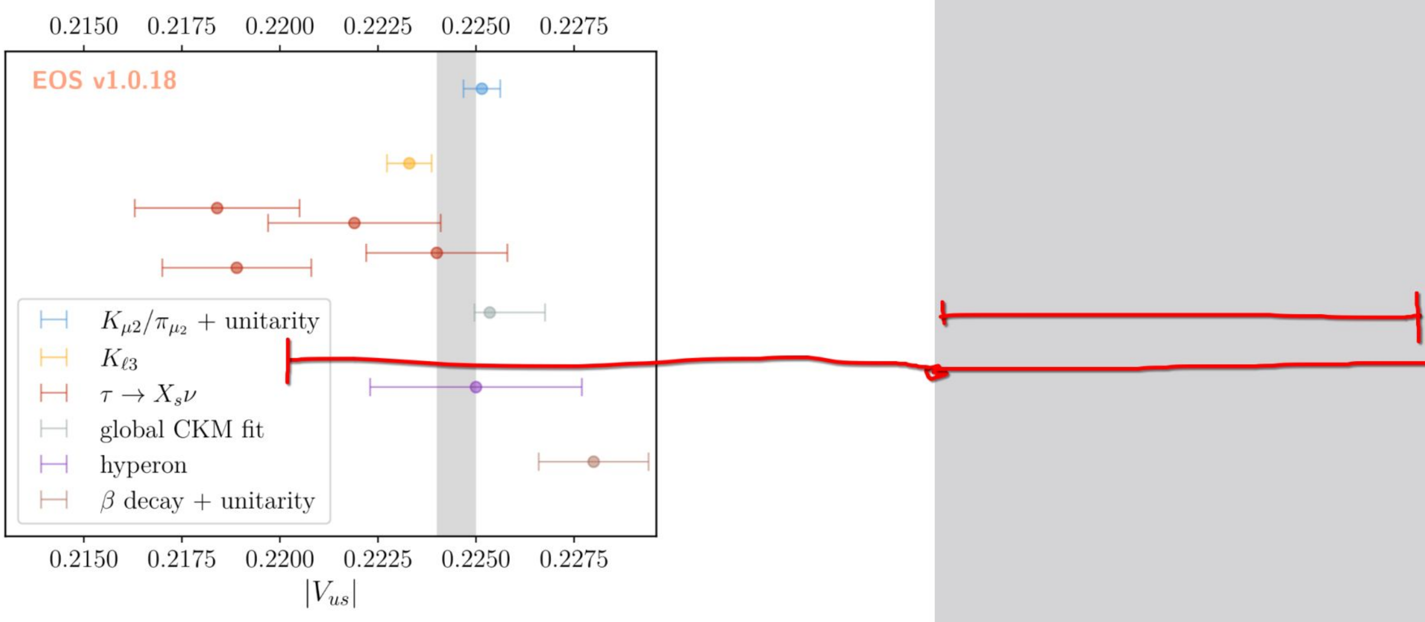
Cabibbo angle anomaly overview

- Hyperon (from hep-ph/0307214)
- CKMfitter



Cabibbo angle anomaly overview

- New LHCb (arXiv:2511.15681) using lattice (arXiv:2507.09970)



Connecting two body and three body

- Callan-Treiman theorem:
  - $f_0(m_K^2 - m_\pi^2) = f_K/f_\pi + \Delta_{\rm CT}$
  - $f_0(m_\pi^2 - m_K^2) = f_\pi/f_K + \widetilde{\Delta_{\rm CT}}$
- Links $K_{\ell 2}$ to $K_{\ell 3}$, $\tau \to X_s \nu$
  - So it's important to fit all three s->u transitions

Cabibbo angle anomaly overview